Multiuser Detection via Compressive Sensing

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***Abstract*—In this paper, we consider a multiuser detection technique when the signal sparsity is changing over time. The key ingredient of our method is a clever switching between the CS reconstruction algorithm and classical detection depending on the sparsity level of the signals being detected. Since none of these approaches is uniformly better in a situation where the sparsity level is varying, proposed switching algorithm can effectively combine the merits of both. We show that the proposed switching algorithm provides substantial performance gain over individual algorithms in the multiuser detection of CDMA downlink.**

***Index Terms*—Compressive sensing, sparsity, orthogonal matching pursuit, multiuser detection.**

I. Introduction

As a paradigm guaranteeing the perfect reconstruction of a sparse signal from a small set of linear measurements, compressive sensing (CS) has received much attention in recent years [1], [2]. While the CS paradigm works well in many signal and image processing applications where the signal vector contains lots of zero elements (such as wavelet transformed natural images), its application on communication systems is rather limited mainly because the information vector being transmitted is far from sparse. Note that the CS reconstruction algorithm generally performs worse than the classic estimation/detection algorithm in the detection of non-sparse signals.

Our aim in this work is to demonstrate that the CS paradigm, in conjunction with the classical detection method, is suitable for multiuser detection. Two key features of the multiuser communication link enabling our hybrid approach are the existence of interferences and variations of interference levels. Since the multiuser communication link is interference limited due to intercell, intracell, and intersymbol interferences [6], the system is typically well modeled as the underdetermined system. This, together with the fact that the number of users is varying over time, frequency, and location motivates our work to exploit the CS reconstruction algorithm. Note that it is now well known that the CS paradigm is suitable, and in fact, beneficial, for detecting sparse signals. Hence, by selectively using the CS reconstruction algorithm when the signal to be detected is sparse, we can improve the link level performance of the multiuser communication system considerably.

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The key ingredient of our method is a clever switching between the CS reconstruction algorithm and classical detection depending on the sparsity level of signals being detected. That is, when the signal is sparse, the CS reconstruction algorithm is employed, and when the signal is non-sparse, classical estimation technique is applied. Since none of these approaches is uniformly better in a situation where the sparsity level is varying, proposed switching algorithm can effectively combine the merits of both.

II. Multiuser Detection using the Compressive Sensing

A. Multiuser System Model

Consider the downlink of CDMA system where the basestation can transmit information to maximally N user terminals (i.e., spreading factor of the system is N). If K user terminals are active ($K \leq N$), then K symbols are modulated via orthogonal codes in the basestation and then transmitted through the fading channels. In this system, M-dimensional received chip vector $\mathbf{y}$ corresponding to one symbol period (N-chip period) is given by

$$\mathbf{y} = \mathbf{\Phi x} + \mathbf{v} \tag{1}$$

where $\mathbf{\Phi} = \mathbf{HW}$ is the composite of the channel matrix $\mathbf{H} \in \mathbb{C}^{M \times N}$ and the orthogonal Walsh matrix $\mathbf{W} \in \mathbb{C}^{N \times N}$. In addition, $\mathbf{x} \in (\mathbb{Q} \cup \{0\})^N \subset \mathbb{Z}^N$ is the symbol vector whose elements are drawn either from a modulation set $\mathbb{Q}$ or 0 and $\mathbf{v}$ is the complex Gaussian noise ($\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$). Note that an element of $\mathbf{x}$ can be zero since some of user terminals is in idle state and hence not transmitting data. This setup is valid in various applications such as the uplink of CDMA systems and wireless sensor networks. At the heart of this model is the sparsity of the input vector $\mathbf{x}$ being detected. Indeed, in many multiuser systems, the number of user terminals being serviced (i.e., active users) K is much smaller than the maximum number of user terminals N so that the sparsity $S = \frac{N-K}{N}$ is very high for a fair amount of service period. Even in the case where $S \approx 0$, by modeling the signal of users close to the basestation as a noise[1], one can still use the system model in (1).

[1]This argument holds true for downlink scenario. Same argument can be applied to the uplink scenario by modeling the signal of users far away from the basestation as a noise.

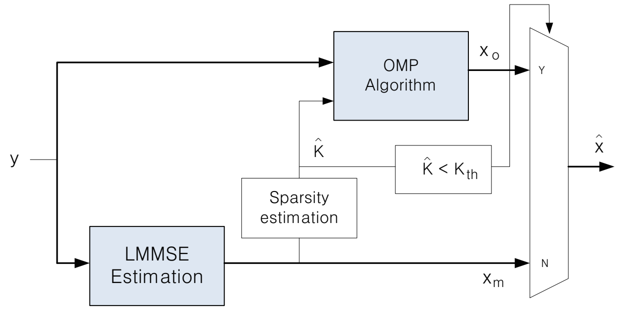

Fig. 1. The proposed multiuser detection method employing the LMMSE estimation and OMP algorithm.

B. Compressive Sensing and Orthogonal Matching Pursuit

Suppose an unknown signal $\mathbf{x} \in \mathbb{C}^N$ to be detected is K-sparse, meaning that there are only K non-zero elements in $\mathbf{x}$. Then the goal of the CS is to recover this sparse vector $\mathbf{x}$ using a measurements $\mathbf{y} \in \mathbb{C}^M$. Usually, the number of measurements M is smaller than the number of signals being detected ($M < N$). The relationship between the measurements $\mathbf{y}$ and the sparse signal $\mathbf{x}$ is

$$\mathbf{y} = \mathbf{\Phi x} \tag{2}$$

Since the system is underdetermined, it is in general impossible to obtain an accurate reconstruction of the original input $\mathbf{x}$ by using a conventional inverse transform of $\mathbf{\Phi}$. Whereas, due to the prior information of signal sparsity, $\mathbf{x}$ can be perfectly reconstructed as long as the transformation matrix $\mathbf{\Phi}$ satisfies the condition derived from the RIP. Note that a sensing matrix $\mathbf{\Phi}$ is said to satisfy the RIP of order K if there exists a constant δ such that [2]

$$(1-\delta)\left\|\mathbf{x}\right\|_2^2 \le \left\|\mathbf{\Phi x}\right\|_2^2 \le (1+\delta)\left\|\mathbf{x}\right\|_2^2 \tag{3}$$

for any K-sparse vector $\mathbf{x}$ ($\|\mathbf{x}\|_0 \le K$). In particular, the minimum of all constants δ satisfying (3) is referred to as the isometry constant δ_K. Over the years, many efforts have been made to find out the condition guaranteeing the exact recovery of sparse signals. For example, if $\delta_{2K} < \sqrt{2}-1$, then the ℓ_1-minimization problem

$$\hat{\mathbf{x}} = \arg\min \|\mathbf{x}\|_1 \text{ subject to } \mathbf{y} = \mathbf{\Phi x} \tag{4}$$

provides perfect reconstruction of $\mathbf{x}$ [3]. Since computational complexity associated with the linear programming (LP) technique to solve (4) is considerable, greedy search algorithms sequentially investigating the support (set of nonzero values) of the sparse signal have been received much attention as cost effective solutions in recent years. Orthogonal matching pursuit (OMP) is a representative method in the greedy algorithm family [4]. In each iteration of the OMP algorithm, correlations between each column of $\mathbf{\Phi}$ and modified measurements (so called residual) are compared and an index of the column generating maximal correlation is chosen as an estimate of the support element (see [4], [5] for details).

C. Switching between OMP and LMMSE

The structure of the proposed multiuser detector is shown in Fig. 1. Key feature of the proposed method is to switch the recovery scheme depending on the sparsity level of an input signal. That is, when the sparsity S is high (i.e., K is small), the OMP algorithm is being used and when S is low, classical detection scheme is employed. In this letter, we only consider the LMMSE detector for the sake of simplicity. As is clear from Fig. 2, the OMP algorithm outperforms the LMMSE detection schemes for highly sparse scenario and vice versa, it is of importance to switch the algorithm depending on the sparsity level.

In obtaining the switching threshold K_{th}, a condition derived from the RIP is employed. A well known result derived from the RIP is that $M \times N$ random matrices with i.i.d. entries from the Gaussian distribution obey the RIP with $\delta_K < \epsilon$ with very high probability if M satisfies

$$M \ge \frac{\rho K \log \frac{N}{K}}{\epsilon^2}. \tag{5}$$

where ρ is a constant. This result is readily applied to the complex Gaussian random matrices. Recent results show that if $\delta_{K+1} < \frac{1}{\sqrt{K}+1}$ then the OMP can perfectly recover K-sparse signals [5]. For a system with $K \gg 1$, which is indeed true for most of multiuser system, we can use an approximated version $\delta_K < \frac{1}{\sqrt{K}}$ and hence

$$M \ge \rho K^2 \log \frac{N}{K}. \tag{6}$$

Since under this condition the OMP algorithm guarantees near perfect reconstruction of $\mathbf{x}$ from $\mathbf{y} = \mathbf{\Phi x}$, the maximum of K becomes the decision threshold K_{th}. That is,

$$K_{th} = \max\{K; M - \rho K^2 \log \frac{N}{K} > 0\}. \tag{7}$$

If the estimated K is smaller than this threshold (i.e., $\hat{K} < K_{th}$), the OMP algorithm is employed and the LMMSE detector is used otherwise. It is worth mentioning that even though (7) is satisfied, exact recovery may not be possible since the model in (1) contains noise. Nevertheless, in our experiments, we observe that $\rho \approx 0.1$ provides fairly good performance.

The number of non-sparse elements $\hat{K}$ used for the algorithm selection can be estimated using the output of the LMMSE detector. Noting that the LMMSE estimate x_i of the i-th user terminal can be modeled as $\hat{x}_i = x_i + w_i$ when the signal is present and $\hat{x}_i = w_i$ when the signal is absent, the hypothesis testing to identify an element of the support is

$$\begin{cases} \mathcal{H}_0: & \hat{x}_i = w \qquad \text{(inactive user)} \\ \mathcal{H}_1: & \hat{x}_i = x_i + w \quad \text{(active user).} \end{cases} \tag{8}$$

Since x_i is an element of modulation set and hence not fixed, we can use generalized likelihood ratio test (GLRT) for identifying elements of the support [7]. Note that the GLRT approach computes maximum likelihood estimates for each hypothesis (only for $\mathcal{H}_1$ in this case) and then use it as a true

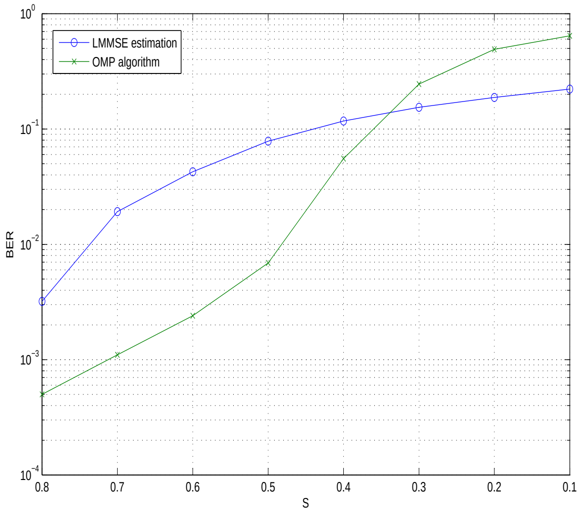

Fig. 2. Performance of the LMMSE detector and OMP algorithm as a function of sparsity S (SNR = 10 dB).

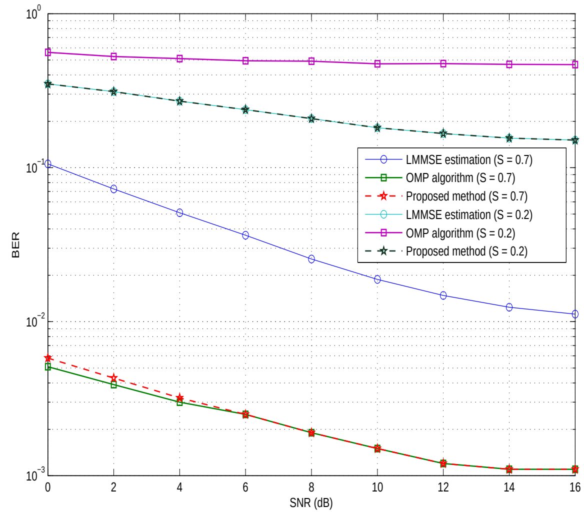

Fig. 3. Performance of the proposed method for $S = 0.7$ and $S = 0.2$.

parameter in the likelihood ratio test (LRT). The likelihood ratio of the GLRT is

$$\frac{P_r(\hat{x}_i|\,\mathcal{H}_1)}{P_r(\hat{x}_i|\,\mathcal{H}_0)} = \frac{\max_{x_i \in \mathbb{Q}} P_r(\hat{x}_i|\,x_i)}{P_r(\hat{x}_i|\,x_i = 0).} \tag{9}$$

After manipulation, the decision rule to choose $\mathcal{H}_1$ is simplified to

$$\Re(\hat{x}_i^* x_{ml}) > \frac{|x_{ml}|^2}{2} + \sigma_w^2 \ln \tau \tag{10}$$

where x_{ml} is the ML estimate of $\hat{x}_i$ ($x_{ml} = \arg\max_{x_i} P(\hat{x}_i|x_i)$) and τ is the predefined threshold.

III. Simulation Results and Discussions

In order to demonstrate the effectiveness of the proposed method, we test the OMP algorithm, LMMSE detector, and the proposed hybrid method in the downlink of CDMA system. We set the maximal number N of user terminals to 128 ($M = 0.9N$) and test the performance for various levels of sparsity S. Channel between the basestation and user terminal is modeled as non-dispersive independent Rayleigh fading and 16-QAM modulation is used for symbol transmission.

In our first experiment, we test two corner cases; highly sparse ($S = 0.7$) and moderately sparse ($S = 0.2$) scenarios. As shown in Fig. 3, in highly sparse scenario, the OMP algorithm outperforms the LMMSE detector, providing more than order of magnitude improvement in BER performance. Also, since the proposed method returns the output of the OMP algorithm, we observe that the performance of the OMP algorithm and proposed method is almost identical. On the contrary, the OMP algorithm exhibits the worst performance in the slightly sparse scenario ($S = 0.2$). In this case, the proposed method follows the performance of the LMMSE detector.

In order to observe the benefit of the proposed method in more realistic scenario, we simulate the proposed method in a situation where the sparsity is randomly varying ($S \in \text{Unif}(0, 1)$). Since the proposed scheme switches to the algorithm providing better performance, it is no surprise that the proposed method shows the best performance. Indeed, as shown in Fig. 4, the proposed method outperforms the OMP and LMMSE detector over whole ranges of simulations. We can conclude that the flexibility of the proposed hybrid scheme lends itself to the usage of various scenarios of multiuser communications.

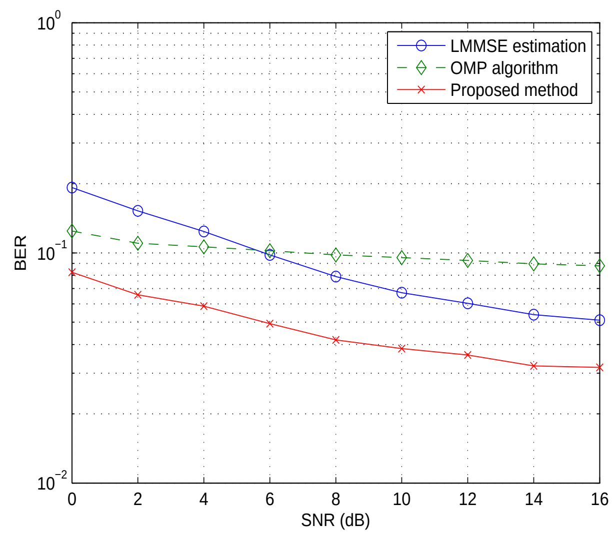

Fig. 4. Performance of the proposed method when S is randomly varying.

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