On the Beamforming Design for MIMO Multi-Pair Two-way Relay Channels

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Abstract

In this paper, we propose a simple yet effective beamforming technique for multiple-input multiple-output (MIMO) multi-pair two-way relay channels. Two key ingredients in our technique are adoption of signal space alignment (SSA) for transmit and receive beamforming and amplify-and-forward (AF) relay beamforming using advanced zero-forcing (ZF) technique. From the sum-rate analysis on MIMO multi-pair two-way relay channels, we show that the proposed method achieves multiplexing gain proportional to the number of user-pairs.

Index Terms

Two-way relay channels, relay beamforming, signal space alignment, sum-rate maximization.

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I. Introduction

Recently, relay-assisted communication systems have been received much attention as a means to enhance coverage, reliability and throughput of the cellular network. In particular, two-way relay systems where nodes exchange information using intermediate relays have been an active research area in the last few years [1]-[3]. Unlike four-phase two-way relay systems which require four time slots to exchange messages between two users, two-way relaying systems require two time slots for transferring informations [1], [6]; In the multiple-access (MAC) phase, two users simultaneously transmit to the relay, and in the broadcast (BC) phase, the relay broadcasts a composite signal to the active users. Since each user knows its own information, back-propagating self-interferences can be eliminated from the received signal, and hence, two-way relay system does not suffer from spectral efficiency loss caused by the pre-log factor of $\frac{1}{2}$ in the half duplex relaying.

In recent years, to further improve the network throughput, two-way relay system has been extended to multi-pair two-way system where multiple user-pairs exchange messages using shared relay(s) [4], [10], and even further to multi-user multi-way system where multiple users exchange messages among themselves [5], [7]. In both scenarios, simultaneously transmitted signals from multiple users and overheard undesired signals from relay(s) incur interference, deteriorating the performance of the system. Therefore, proper control of the inter-user interference is crucial to ensure a reliable communication of multi-pair relay systems. In fact, in the multi-pair two-way relay channel, the main bottleneck on the system performance is the interference from other user-pairs. In [9], multi-pair two-way relay channel has been investigated where K-pairs of single antenna users communicate bi-directionally via a single N antenna relay. In this channel, the relay should have more than $2K$ antennas for achieving multiplexing gain of K.

In this paper, we propose a simple beamforming technique for the MIMO multi-pair two-way relay system. Key ingredient in our approach is a combination of transmit and receive beamforming and the AF relay beamforming for controlling the inter-user interference. While the purpose of transmit and receive beamforming is to align signals coming from each user-pair for reducing the effective number of interferences, the purpose of the relay beamforming is to control the noise enhancement by searching the relay beamforming vector maximizing the sum-rate. Toward this end, two AF relay beamforming schemes are proposed; the enhanced ZF (E-ZF) and the iterative AF (I-AF) relay beamforming. The E-ZF scheme exploits the selection diversity by choosing a solution among multiple candidates satisfying ZF criterion and the I-AF searches the optimal beamforming vector in an iterative fashion.

The rest of this paper is organized as follows. In Section II, we briefly review the MIMO multi-pair two-way relay channel. In Section III and IV, we present the transmit/receive beamforming based on the SSA technique and the AF relay beamforming schemes. We present the sum-rate performance in Section V and we conclude our paper in Section VI.

II. System Model

As illustrated in Fig. 1, MIMO multi-pair two-way relay channel consists of $2K$ users having M antennas and a relay having N antennas. In this channel, a user i exchanges a message with its partner $\pi(i)$ using two consecutive time slots where $\pi(i)$ is an index function indicating the partner of user i. We assume that the relay has the perfect channel state information (CSI) of all links and each user node knows the channel between the relay and itself and the relay and its partner. Further, we assume that there is no direct link between user-pairs so that no overheard side information among users is possible.

In the first time slot (i.e., MAC phase), the transmit signal from user i to the relay is

$$\mathbf{x}_i = \mathbf{v}_i s_i, \quad i \in \{1, \cdots, 2K\},$$

where s_i is data symbol and $\mathbf{v}_i$ is a unitary beamforming vector satisfying the transmit power constraint $\left(\mathbb{E}\left\{\mathrm{tr}\left[\mathbf{x}_i\mathbf{x}_i^H\right]\right\} \leq P_i\right)$. Then the received signal vector at the relay becomes

$$\mathbf{y}_r = \sum_{i=1}^{2K} \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_r, \tag{1}$$

where the entries of $\mathbf{H}_i$, the uplink channel matrix from user i to the relay node, are i.i.d. complex Gaussian random variables with $\mathcal{CN}(0,1)$ and $\mathbf{n}_r$ is the $N \times 1$ complex noise vector whose components are i.i.d. with a circularly symmetric complex Gaussian with $\mathcal{CN}(0,\sigma_r^2)$. Once $\mathbf{y}_r$ is received, the relay generates the transmit signal $\mathbf{x}_r$ as

$$\mathbf{x}_r = \gamma \mathbf{W} \mathbf{y}_r, \tag{2}$$

where $\mathbf{W} \in \mathbb{C}^{N \times N}$ is the relay beamforming matrix and γ is the power normalizing coefficient $\left(\mathbb{E}\left\{\mathrm{tr}\left[\mathbf{x}_r\mathbf{x}_r^H\right]\right\} \leq P_r\right)$. In the second time slot (i.e., BC phase), the relay broadcasts $\mathbf{x}_r$ to all user nodes and hence the received signal $\mathbf{y}_{\pi(i)}$ at user $\pi(i)$ becomes

$$\begin{aligned} \mathbf{y}_{\pi(i)} &= \mathbf{G}_{\pi(i)}\mathbf{x}_r + \mathbf{n}_{\pi(i)} \\ &= \gamma \mathbf{G}_{\pi(i)}\mathbf{W}\left(\sum_{i=1}^{2K}\mathbf{H}_i\mathbf{x}_i + \mathbf{n}_r\right) + \mathbf{n}_{\pi(i)}, \end{aligned} \tag{3}$$

where $\mathbf{G}_{\pi(i)} \in \mathbb{C}^{M \times N}$ is the downlink channel matrix from the relay node to user $\pi(i)$[1] and $\mathbf{n}_{\pi(i)}$ is the $M \times 1$ complex noise vector whose components are i.i.d. with a circularly symmetric complex Gaussian with $\mathcal{CN}(0, \sigma^2_{\pi(i)})$. Since user $\pi(i)$ knows its own transmit signal, back-propagating self-interference $\gamma\mathbf{G}_{\pi(i)}\mathbf{W}\mathbf{H}_{\pi(i)}\mathbf{v}_{\pi(i)}s_{\pi(i)}$ can be removed from $\mathbf{y}_{\pi(i)}$. The modified received signal $\tilde{\mathbf{y}}_{\pi(i)}$ of $\pi(i)$ after canceling this self-interference signal is

$$\begin{aligned} \tilde{\mathbf{y}}_{\pi(i)} &= \mathbf{y}_{\pi(i)} - \gamma\mathbf{G}_{\pi(i)}\mathbf{W}\mathbf{H}_{\pi(i)}\mathbf{v}_{\pi(i)}s_{\pi(i)} \\ &= \gamma\mathbf{G}_{\pi(i)}\mathbf{W}\left(\mathbf{H}_i\mathbf{v}_i s_i + \sum_{j=1, j\neq i,\pi(i)}^{2K}\mathbf{H}_j\mathbf{v}_j s_j\right) + \tilde{\mathbf{n}}_{\pi(i)}, \end{aligned} \tag{4}$$

where $\tilde{\mathbf{n}}_{\pi(i)} = \gamma\mathbf{G}_{\pi(i)}\mathbf{W}\mathbf{n}_r + \mathbf{n}_{\pi(i)}$. Once the self interference is removed, a receive beamforming vector $\mathbf{u}^H_{\pi(i)}$ is applied to $\tilde{\mathbf{y}}_{\pi(i)}$ for generating the estimate of the desired symbol s_i. The estimated symbol of user i is given by

$$\tilde{s}_i = \gamma\mathbf{u}^H_{\pi(i)}\mathbf{G}_{\pi(i)}\mathbf{W}\left(\mathbf{H}_i\mathbf{v}_i s_i + \sum_{j=1, j\neq i,\pi(i)}^{2K}\mathbf{H}_j\mathbf{v}_j s_j\right) + \mathbf{u}^H_{\pi(i)}\tilde{\mathbf{n}}_{\pi(i)}. \tag{5}$$

III. Beamforming with Signal Space Alignment (SSA)

In this section, we provide the transmit and receive beamforming scheme based on the SSA [7]. The beamforming scheme with SSA aligns channel direction of each user-pairs both in the uplink channel (MAC phase) and the downlink channel (BC phase). While achieving the same multiplexing gain, this beamforming scheme enables each user-pairs to control interferences from other user-pairs with minimal number of relay antennas. In fact, we show that the multiplexing gain of K is achieved with only half number of relay antennas in [9].

[1]Under the assumption that channel is invariant during two time slots, $\mathbf{G}_i$ would be $\mathbf{H}_i^H$.

A. Transmit and Receive Beamforming

Key feature of the transmit beamforming is to exploit SSA for aligning the uplink channel direction of user i and its partner $\pi(i)$. As a result, the number of channels experienced by the relay is reduced half (from $2K$ to K). In a similar way, we also use the SSA in the receive beamforming so that we can align downlink channel by user-pair and make interference-free communication.

First, transmit beamforming vectors of each user-pair ($\mathbf{v}_i$ and $\mathbf{v}_{\pi(i)}$) are designed such that the following alignment condition is satisfied

$$\mathrm{span}\left(\mathbf{H}_i \mathbf{v}_i\right) = \mathrm{span}\left(\mathbf{H}_{\pi(i)} \mathbf{v}_{\pi(i)}\right), \tag{6}$$

where $\mathrm{span}\left(\mathbf{A}\right)$ is the column space of $\mathbf{A}$. Simple solution satisfying (6) is given by [8],

$$\underbrace{\left[\begin{array}{cc} \mathbf{H}_i & -\mathbf{H}_{\pi(i)} \end{array}\right]}_{N \times 2M} \left[\begin{array}{c} \tilde{\mathbf{v}}_i \\ \tilde{\mathbf{v}}_{\pi(i)} \end{array}\right] = \mathbf{0}, \tag{7}$$

where $\tilde{\mathbf{v}}_i = \|\tilde{\mathbf{v}}_i\| \cdot \mathbf{v}_i$ and $\tilde{\mathbf{v}}_{\pi(i)} = \|\tilde{\mathbf{v}}_{\pi(i)}\| \cdot \mathbf{v}_{\pi(i)}$, respectively, and $\mathbf{v}_i$ and $\mathbf{v}_{\pi(i)}$ are the normalized unitary transmit beamforming vectors. Since columns of $\mathbf{H}_i$ and $\mathbf{H}_{\pi(i)}$ are assumed to be independent, $\mathbf{v}_i$ and $\mathbf{v}_{\pi(i)}$ satisfying (6) can be found as long as $N < 2M$ (i.e., when the nullspace of $\left[\begin{array}{cc} \mathbf{H}_i & -\mathbf{H}_{\pi(i)} \end{array}\right]$ is nonempty).

In a similar way, user i and $\pi(i)$ in the downlink channel (BC phase) can cooperatively construct the receive beamforming vectors satisfying the following alignment condition[2]

$$\mathrm{span}\left(\mathbf{u}_i^H \mathbf{G}_i\right) = \mathrm{span}\left(\mathbf{u}_{\pi(i)}^H \mathbf{G}_{\pi(i)}\right). \tag{8}$$

Unitary receive beamforming vector $\mathbf{u}_i$ and $\mathbf{u}_{\pi(i)}$ satisfying (8) can be obtained by

$$\left[\begin{array}{cc} \tilde{\mathbf{u}}_i^H & \tilde{\mathbf{u}}_{\pi(i)}^H \end{array}\right] \underbrace{\left[\begin{array}{c} \mathbf{G}_i \\ -\mathbf{G}_{\pi(i)} \end{array}\right]}_{2M \times N} = \mathbf{0}, \tag{9}$$

[2]If the uplink and downlink channels are reciprocal (TDD), $\mathbf{G}_i = \mathbf{H}_i^H$ and hence the feedback of $\mathbf{H}$ from the relay to the user node is unnecessary. In other words, each user nodes can design transmit and receive filter $\left(\mathbf{u}_i^H = \mathbf{v}_i\right)$ using the pilot signal sent from the relay node and no feedback of uplink channel information $\mathbf{H}_i$ is required.

where $\tilde{\mathbf{u}}_i = \|\tilde{\mathbf{u}}_i\| \cdot \mathbf{u}_i$ and $\tilde{\mathbf{u}}_{\pi(i)} = \|\tilde{\mathbf{u}}_{\pi(i)}\| \cdot \mathbf{u}_{\pi(i)}$, respectively, and $\mathbf{u}_i$ and $\mathbf{u}_{\pi(i)}$ are the normalized unitary receive beamforming vectors.

From (7) and (9), we have

$$\frac{\mathbf{H}_i\mathbf{v}_i}{\|\mathbf{H}_i\mathbf{v}_i\|_2} = \frac{\mathbf{H}_{\pi(i)}\mathbf{v}_{\pi(i)}}{\|\mathbf{H}_{\pi(i)}\mathbf{v}_{\pi(i)}\|_2}, \quad \frac{\mathbf{u}_i^H\mathbf{G}_i}{\|\mathbf{u}_i^H\mathbf{G}_i\|_2} = \frac{\mathbf{u}_{\pi(i)}^H\mathbf{G}_{\pi(i)}}{\|\mathbf{u}_{\pi(i)}^H\mathbf{G}_{\pi(i)}\|_2}.$$

Denoting $\bar{\mathbf{h}}_{i,\pi(i)} = \frac{\mathbf{H}_i\mathbf{v}_i}{\|\mathbf{H}_i\mathbf{v}_i\|_2}$ and $\bar{\mathbf{g}}_{i,\pi(i)}^H = \frac{\mathbf{u}_i^H\mathbf{G}_i}{\|\mathbf{u}_i^H\mathbf{G}_i\|_2}$, (5) can be rewritten as

$$\tilde{s}_i = \gamma\beta_{\pi(i)}\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}\left(\alpha_i\bar{\mathbf{h}}_{i,\pi(i)}s_i + \sum_{j=1,j\neq i,\pi(i)}^{2K}\alpha_j\bar{\mathbf{h}}_{j,\pi(j)}s_j\right) + \mathbf{u}_{\pi(i)}^H\tilde{\mathbf{n}}_{\pi(i)}, \tag{10}$$

where $\alpha_i = \|\mathbf{H}_i\mathbf{v}_i\|_2$ and $\beta_{\pi(i)} = \|\mathbf{u}_{\pi(i)}^H\mathbf{G}_{\pi(i)}\|_2$ are effective channel gains from user i to the relay and from the relay to user $\pi(i)$, respectively. Note that, as a result of the alignment in the uplink and downlink channels, user $\pi(i)$ receives the desired data stream and only $K-1$ independent interference streams. The achievable sum rate of the aligned system in (10) becomes

$$R_{sum}(\mathbf{W}) = \sum_{i=1}^{2K}\frac{1}{2}\log_2\left(1+\frac{\rho\gamma^2\alpha_i^2\beta_{\pi(i)}^2\left|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}\bar{\mathbf{h}}_{i,\pi(i)}\right|^2}{\rho\gamma^2\beta_{\pi(i)}^2\sum_{j=1,j\neq i,\pi(i)}^{2K}\alpha_j^2\left|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}\bar{\mathbf{h}}_{j,\pi(j)}\right|^2+\gamma^2\beta_{\pi(i)}^2\left\|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}\right\|_2^2+1}\right). \tag{11}$$

B. Multiplexing Gain

In this subsection, we investigate the achievable multiplexing gain of the MIMO K-pair two-way relay channel employing SSA-based transmit and receive beamforming. Since the relay beamforming matrix $\mathbf{W}$ in (11) needs to be specified to evaluate the multiplexing gain, we use the conventional ZF scheme annihilating inter-user interferences in our analysis. The achievable multiplexing gain is defined as

$$\eta = \lim_{\rho\to\infty}\frac{R_{sum}(\mathbf{W})}{\log_2(\rho)}. \tag{12}$$

Our main result is formally stated in the following theorem.

Theorem 1: *For the MIMO multi-pair two-way relay channel where $2K$ users have M antennas and a shared relay has N antennas, the achievable multiplexing gain of the SSA based beamforming becomes K as long as $N \geq K$.*

Proof: Under the constraint $N \leq 2M$ to make the SSA feasible, the relaying beamforming matrix $\mathbf{W}_{ZF}$ is designed such that all inter-user interferences in (11) become zero, i.e.,

$$\begin{aligned}\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}_{ZF}\bar{\mathbf{h}}_{j,\pi(j)} &= 0 \quad \text{if} \quad i \neq j,\\ \bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}_{ZF}\bar{\mathbf{h}}_{i,\pi(i)} &= 1 \quad \text{if} \quad i = j.\end{aligned} \tag{13}$$

In other words,

$$\bar{\mathbf{G}}\mathbf{W}_{ZF}\bar{\mathbf{H}} = \mathbf{I}_K, \tag{14}$$

where $\bar{\mathbf{G}} = [\bar{\mathbf{g}}_{1,K+1} \cdots \bar{\mathbf{g}}_{K,2K}]^H \in \mathbb{C}^{K\times N}$ and $\bar{\mathbf{H}} = \left[\bar{\mathbf{h}}_{1,K+1} \cdots \bar{\mathbf{h}}_{K,2K}\right] \in \mathbb{C}^{N\times K}$ are the effective uplink and downlink channel matrix, respectively. Under the assumption that $N \geq K$, $\mathbf{W}_{ZF}$ satisfying (14) becomes

$$\mathbf{W}_{ZF} = \bar{\mathbf{G}}^{\dagger}\bar{\mathbf{H}}^{\dagger} \tag{15}$$

where $\mathbf{A}^{\dagger}$ denotes pseudo-inverse of matrix $\mathbf{A}$. Plugging $\mathbf{W}_{ZF} = \bar{\mathbf{G}}^{H\dagger}\bar{\mathbf{H}}^{\dagger}$ into (11), we have

$$R_{sum}^{ZF}(\mathbf{W}_{ZF}) = \frac{1}{2}\sum_{i=1}^{2K}\log_2\left(1 + \frac{\rho\gamma^2\alpha_i^2\beta_{\pi(i)}^2\left|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}_{ZF}\bar{\mathbf{h}}_{i,\pi(i)}\right|^2}{\gamma^2\beta_{\pi(i)}^2\left\|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}_{ZF}\right\|_2^2 + 1}\right), \tag{16}$$

where γ is defined as

$$\gamma = \sqrt{\frac{\rho}{\sum_{i=1}^{2K}\rho\alpha_i^2\mathrm{tr}\left(\mathbf{W}_{ZF}\bar{\mathbf{h}}_{i,\pi(i)}\bar{\mathbf{h}}_{i,\pi(i)}^H\mathbf{W}_{ZF}^H\right) + \mathrm{tr}\left(\mathbf{W}_{ZF}\mathbf{W}_{ZF}^H\right)}}. \tag{17}$$

Denoting $c_i = \frac{\gamma^2\alpha_i^2\beta_{\pi(i)}^2\left|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}_{ZF}\bar{\mathbf{h}}_{i,\pi(i)}\right|_2^2}{\gamma^2\beta_{\pi(i)}^2\|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}_{ZF}\|_2^2+1}$, (16) can be rewritten as $R_{sum}^{ZF}(\mathbf{W}_{ZF}) = \frac{1}{2}\sum_{i=1}^{2K}\log_2(1+\rho c_i)$. Noting that $c_i > 0$ (since $\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}_{ZF}\bar{\mathbf{h}}_{i,\pi(i)} = 1$), the multiplexing gain becomes

$$\begin{aligned}\eta &= \lim_{\rho\to\infty}\frac{R_{sum}^{ZF}(\mathbf{W}_{ZF})}{\log_2(\rho)} \\ &= \lim_{\rho\to\infty}\frac{\frac{1}{2}\sum_{i=1}^{2K}\log_2(1+c_i\rho)}{\log_2(\rho)} = K,\end{aligned} \tag{18}$$

which completes the proof of the theorem. ■

It is worth mentioning that if the SSA scheme is not applied, the relay will receive $2K$ independent channels and hence requires $N \geq 2K$ antennas for achieving multiplexing gain of K. Whereas, in the proposed method, each user-pair can experience the same spatial direction due to the SSA. Therefore, as long as $N \geq K$, multiplexing gain of K can be obtained.

IV. AF Relay Beamforming Designs

Since the sum-rate in (11) is a function of the beamforming matrix $\mathbf{W}$, proper design of $\mathbf{W}$ is of great importance for improving performance for MIMO multi-pair two-way relay systems. In this section, we propose two AF relay beamforming strategies for achieving additional (beamforming) gain over the simple ZF scheme in (15). Our first method, referred to as enhanced ZF (E-ZF) scheme, opportunistically selects a solution among multiple ZF candidates in the null space of effective interference channel matrix. The second method, referred to as iterative AF (I-AF) scheme, iteratively searches a local optimal solution for $\mathbf{W}$.

A. Enhanced ZF (E-ZF) Relaying Scheme

As mentioned, when the number of relay antennas is larger than the number of user-pairs $(N \geq K)$, all inter-user interferences can be removed using the ZF-based relay beamforming. The well-known drawback of the ZF technique is the sum-rate loss at the low and mid SNR regime. Our proposed E-ZF alleviates the sum-rate loss using the selection diversity while maintaining the benefit (interference annihilation) of the ZF technique. Specifically, by exploiting the property of the kronecker product, we create multiple candidates, among which we opportunistically select one achieving maximal beamforming gain.

Employing the property of the kronecker product, $\text{vec}\left(\mathbf{AXB}\right) = \left(\mathbf{B}^T \otimes \mathbf{A}\right) \text{vec}\left(\mathbf{X}\right)$[3] [13], the zero-interference condition in (14) can be expressed as

$$\left(\bar{\mathbf{h}}_{j,\pi(j)}^T \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H\right) \text{vec}\left(\mathbf{W}\right) = 0, \quad i,j \in \{1,2,\ldots,K\} \quad \text{and} \quad i \neq j, \tag{19}$$

where $\otimes$ denotes the kronecker product. Denoting $\mathbf{u}_{i,j} = \left(\bar{\mathbf{h}}_{j,\pi(j)}^T \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H\right)$ and $\mathbf{w} = \text{vec}\left(\mathbf{W}\right)$, (19) becomes

$$\mathbf{u}_{i,j}\mathbf{w} = 0, \quad i,j \in \{1,2,\ldots,K\} \quad \text{and} \quad i \neq j. \tag{20}$$

Note that $\mathbf{u}_{i,j} \in \mathbb{C}^{1\times N^2}$ is a vector representing an effective interference channel from user-pair $(j,\pi(j))$ to user-pair $(i,\pi(i))$. By stacking up the effective interference vectors, the zero-interference condition can be obtained as

$$\mathbf{U}\mathbf{w} = \begin{bmatrix} \mathbf{u}_{1,2} \\ \vdots \\ \mathbf{u}_{K,K-1} \end{bmatrix} \mathbf{w} = \mathbf{0}, \tag{21}$$

where $\mathbf{U} \in \mathbb{C}^{K(K-1)\times N^2}$. Since $\mathbf{U}$ is spanned by $\mathbf{u}_{i,j}$ vectors $(i,j \in \{1,2,\ldots,K\}$ and $i \neq j)$, the rank of $\mathbf{U}$ is $K(K-1)$ and hence the rank of $\text{N}\left(\mathbf{U}\right)$ (i.e., nullspace of $\mathbf{U}$) is $N^2 - K(K-1)$.

[3] $\text{vec}\left(\mathbf{X}\right)$ denotes a linear transformation vectorizing the matrix $\mathbf{X}$ into a column vector $\mathbf{x}$. In our case, $\mathbf{X}$ is an $N \times N$ matrix and $\mathbf{x}$ is an $N^2 \times 1$ vector.

Thus, among the $N^2 - K(K-1)$ nullspace and $\mathbf{W}_{ZF}$ in (15), we opportunistically select the relay beamforming matrix $\mathbf{W}$ maximizing $R_{sum}(\mathbf{w})$ as

$$\mathbf{w}^* = \arg \max_{\mathbf{w} \in \mathrm{N}(\mathbf{U}) \cup \{\mathbf{W}_{ZF}\}} R_{sum}(\mathbf{w}). \tag{22}$$

We note that although any linear combination of $N^2 - K(K-1)$ candidates satisfies (21), the gain in sum-rate by choosing $\mathbf{w}$ among larger candidates is small.

Once $\mathbf{w}^*$ is obtained, we can evaluate the sum-rate by devectorizing it to the beamforming matrix $\mathbf{W}$ as

$$R_{sum}(\mathbf{W}) = \frac{1}{2}\sum_{i=1}^{2K} \log_2\left(1 + \frac{\rho\gamma^2\alpha_i^2\beta_{\pi(i)}^2 \left|\bar{\mathbf{g}}_{i,\pi(i)}^H \mathbf{W}\bar{\mathbf{h}}_{i,\pi(i)}\right|^2}{\gamma^2\beta_{\pi(i)}^2 \left\|\bar{\mathbf{g}}_{i,\pi(i)}^H \mathbf{W}\right\|_2^2 + 1}\right), \tag{23}$$

where γ is the normalization factor given by

$$\gamma = \sqrt{\frac{\rho}{\sum_{i=1}^{2K} \rho\alpha_i^2 \mathrm{tr}\left(\mathbf{W}\bar{\mathbf{h}}_{i,\pi(i)}\bar{\mathbf{h}}_{i,\pi(i)}^H \mathbf{W}^H\right) + \mathrm{tr}\left(\mathbf{W}\mathbf{W}^H\right)}}. \tag{24}$$

In a nutshell, the E-ZF improves $R_{sum}(\mathbf{w})$ by choosing the relay beamforming matrix $\mathbf{W}$ among $N^2 - K(K-1)$ candidates satisfying zero-interference condition. For example, if $N = 4$ and $K = 3$, then the beamforming vector $\mathbf{w}$ is chosen among 10 candidates. The process of the E-ZF relay beamforming algorithm is summarized in Table I.

B. Iterative AF (I-AF) Relaying Scheme

The goal of the I-AF relay beamforming scheme is to maximize the sum-rate using a line-search based iterative algorithm. Using the kronecker product operator, $\mathrm{vec}\left(\mathbf{AXB}\right) = \left(\mathbf{B}^T \otimes \mathbf{A}\right)\mathrm{vec}\left(\mathbf{X}\right)$, $R_{sum}(\mathbf{w})$ can be expressed as

$$R_{sum}(\mathbf{w}) = \sum_{i=1}^{2K} \frac{1}{2}\log_2\left(1 + \rho\frac{\mathbf{w}^H\mathbf{Y}_i\mathbf{w}}{\mathbf{w}^H\mathbf{X}_i\mathbf{w}}\right), \tag{25}$$

where

$$\begin{aligned}
\mathbf{Y}_i &= \beta_{\pi(i)}^2\alpha_i^2(\bar{\mathbf{h}}_{i,\pi(i)}^T \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H)^H(\bar{\mathbf{h}}_{i,\pi(i)}^T \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H) \in \mathbb{C}^{N^2 \times N^2} \\
\mathbf{X}_i &= \rho\beta_{\pi(i)}^2 \sum_{j=1, j\neq i,\pi(i)}^{2K} \alpha_j^2(\bar{\mathbf{h}}_{j,\pi(j)}^T \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H)^H(\bar{\mathbf{h}}_{j,\pi(j)}^T \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H) + \beta_{\pi(i)}^2(\mathbf{I} \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H)^H(\mathbf{I} \otimes \bar{\mathbf{g}}_{i,\pi(i)}^H) \\
&\quad + \sum_{i=1}^{2K} \alpha_i^2(\bar{\mathbf{h}}_{i,\pi(i)}^T \otimes \mathbf{I})^H(\bar{\mathbf{h}}_{i,\pi(i)}^T \otimes \mathbf{I}) + \frac{1}{\rho}(\mathbf{I} \otimes \mathbf{I})^H(\mathbf{I} \otimes \mathbf{I}) \in \mathbb{C}^{N^2 \times N^2}.
\end{aligned}$$

Since both the numerator and denominator of $\frac{\mathbf{w}^H\mathbf{Y}_i\mathbf{w}}{\mathbf{w}^H\mathbf{X}_i\mathbf{w}}$ contain $\mathbf{w}$, direct optimization of $R_{sum}(\mathbf{w})$ is very difficult task. In order to convert this into an amenable form, we generate an approximation of (25) in high SNR and low SNR regimes. In a high SNR regime, (25) can be approximated to

$$\lim_{\rho\to\infty} R^H_{sum}(\mathbf{w}) \;=\; \frac{1}{2}\log_2\left(\rho^{2K}\prod_{i=1}^{2K}\frac{\mathbf{w}^H\mathbf{Y}_i\mathbf{w}}{\mathbf{w}^H\mathbf{X}_i\mathbf{w}}\right) \tag{26}$$

and, in a low SNR regime, (25) can be approximated to

$$\lim_{\rho\to 0} R^L_{sum}(\mathbf{w}) \;=\; \frac{1}{2}\log_2\left(1+\rho\sum_{i=1}^{2K}\frac{\mathbf{w}^H\mathbf{Y}_i\mathbf{w}}{\mathbf{w}^H\mathbf{X}_i\mathbf{w}}\right). \tag{27}$$

Let $J_h(\mathbf{w}) = 2^{2R^H_{sum}}$ and $J_l(\mathbf{w}) = 2^{2R^L_{sum}}$, then the optimization problem to find the beamforming vector maximizing both $J_h(\mathbf{w})$ and $J_l(\mathbf{w})$ becomes

$$\mathbf{w}^* = \arg\max_{\mathbf{w}}\left(1+\rho\sum_{i=1}^{2K}\frac{\mathbf{w}^H\mathbf{Y}_i\mathbf{w}}{\mathbf{w}^H\mathbf{X}_i\mathbf{w}} + \rho^{2K}\prod_{i=1}^{2K}\frac{\mathbf{w}^H\mathbf{Y}_i\mathbf{w}}{\mathbf{w}^H\mathbf{X}_i\mathbf{w}}\right). \tag{28}$$

Since this objective function is neither convex nor concave function with respect to $\mathbf{w}$, it is in general very hard to find out the solution of this problem. As a heuristic yet effective method to solve this unconstrained optimization problem, we employ a line-search based gradient ascent algorithm [12]. Let $C(\mathbf{w}) = J_h(\mathbf{w})+J_l(\mathbf{w})$, then the gradient ascent algorithm iteratively updates $\mathbf{w}_k$ such that $C(\mathbf{w}_k) > C(\mathbf{w}_{k-1})$ and hence the updated vector $\mathbf{w}_k$ becomes

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \delta\triangledown C(\mathbf{w}_{k-1}), \quad k = 1.2.\cdots, \tag{29}$$

where δ is the step size of the update and $\triangledown C(\mathbf{w})$ is the gradient of the objective function. In determining the step size, we employ the fast line search scheme so called *Armijo's rule* [12]. In order to improve the convergence speed, the step size of the Armijo's rule is chosen to be $\delta = \nu^m$ where m is the smallest integer such that

$$C(\mathbf{w}_k + \nu^m\cdot\triangledown C(\mathbf{w}_k)) \geq C(\mathbf{w}_k) + \mu\nu^m\mathrm{tr}\left(\triangledown C(\mathbf{w}_k)^H\triangledown C(\mathbf{w}_k)\right), \quad \mu,\nu\in[0,1], \tag{30}$$

where $\triangledown C(\mathbf{w})$ is

$$\begin{aligned}\triangledown C(\mathbf{w}) &= \triangledown J_l(\mathbf{w}) + \triangledown J_h(\mathbf{w})\\ &= \rho\sum_{i=1}^{2K}\frac{2\mathbf{Y}_i\mathbf{w}\left(\mathbf{w}^H\mathbf{X}_i\mathbf{w}\right) - 2\mathbf{X}_i\mathbf{w}\left(\mathbf{w}^H\mathbf{Y}_i\mathbf{w}\right)}{\left(\mathbf{w}^H\mathbf{X}_i\mathbf{w}\right)^2} + \\ &\rho^{2K}\frac{2\left[\sum_{i=1}^{2K}\Big(\prod_{j=1,j\neq i}^{2K}\mathbf{w}^H\mathbf{Y}_j\mathbf{w}\Big)\Big(\prod_{i=1}^{2K}\mathbf{w}^H\mathbf{X}_i\mathbf{w}\Big)\mathbf{Y}_i - \Big(\prod_{j=1,j\neq i}^{2K}\mathbf{w}^H\mathbf{X}_j\mathbf{w}\Big)\Big(\prod_{i=1}^{2K}\mathbf{w}^H\mathbf{Y}_i\mathbf{w}\Big)\mathbf{X}_i\right]\mathbf{w}}{\prod_{i=1}^{2K}\left(\mathbf{w}^H\mathbf{X}_i\mathbf{w}\right)^2}.\end{aligned} \tag{31}$$

Note that the update of beamforming vector is performed by a linear combination of two gradients, $\triangledown J_l\left(\mathbf{w}_{k-1}\right)$ and $\triangledown J_h\left(\mathbf{w}_{k-1}\right)$ so that the gradient $\triangledown J_h\left(\mathbf{w}_{k-1}\right)$ plays a dominant role in the high SNR regime and the gradient $\triangledown J_l\left(\mathbf{w}_{k-1}\right)$ serves a key role in the low SNR regime. In fact, the relay beamforming matrix is automatically adjusted depending on the SNR condition so that the algorithm provides fairly good sum-rate performance in both high and low SNR regimes.

After devectorizing the final output $\mathbf{w}^*$ in to $\mathbf{W}$, we have

$$R_{sum}(\mathbf{W}) = \sum_{i=1}^{2K} \frac{1}{2}\log_2\left(1+\frac{\rho\gamma^2\alpha_i^2\beta_{\pi(i)}^2\left|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}\bar{\mathbf{h}}_{i,\pi(i)}\right|^2}{\rho\gamma^2\beta_{\pi(i)}^2\sum_{j=1,j\neq i,\pi(i)}^{2K}\alpha_j^2\left|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}\bar{\mathbf{h}}_{j,\pi(j)}\right|^2+\gamma^2\beta_{\pi(i)}^2\left\|\bar{\mathbf{g}}_{i,\pi(i)}^H\mathbf{W}\right\|_2^2+1}\right). \quad (32)$$

where γ is the normalization factor given by

$$\gamma = \sqrt{\frac{\rho}{\sum_{i=1}^{2K}\rho\alpha_i^2\mathrm{tr}\left(\mathbf{W}\bar{\mathbf{h}}_{i,\pi(i)}\bar{\mathbf{h}}_{i,\pi(i)}^H\mathbf{W}^H\right)+\mathrm{tr}\left(\mathbf{W}\mathbf{W}^H\right)}}. \quad (33)$$

Although the iterative process we described cannot guarantee the convergence to the global optimal solution due to the non-convexity of the function, by performing multiple runs with distinct initial vectors obtained from random codebook generation, we can improve the sum-rate (see Section V for details). The total process of the iterative algorithm is summarized in Table II.

C. Comments on Complexity

In this subsection, we discuss the complexity of the proposed schemes along with the conventional ZF and iterative Tx beamforming. We measure the complexity of the algorithms by counting the number of floating point operations (flops) [13]. Table III summarizes the number of flops of the proposed methods, conventional ZF, and iterative Tx beamforming [10]. We observe that the flops of iterative Tx beamforming and conventional ZF is equal for $K = 3$ since the dimension of the nullspace is 1 for both schemes. On the other hand, when $K = 6$, the conventional ZF has a clear benefit in complexity over the iterative Tx beamforming. We also observe that additional complexity of the E-ZF with SSA over iterative Tx beamforming are 27% (for $K = 3$) and 30% (for $K = 6$), respectively. The I-AF with SSA achieves the highest computational complexity since the computations associated with (31) are substantial.

V. Simulations and Discussions

In this section, we investigate the sum-rate performance of the proposed E-ZF and I-AF schemes on the MIMO multi-pair two-way relay channel. In our simulation, we set $P_i = P_r = P$ and $\sigma_i^2 = \sigma_r^2 = \sigma^2$ so that the system SNR becomes $\frac{P}{\sigma^2}$. For obtaining the comprehensive picture, we compare the proposed methods with naive AF ($\mathbf{W} = \mathbf{I}_N$) relay beamforming [1], iterative AF relay beamforming [6], and also iterative Tx beamforming [10].

In Fig. 2, we plot the convergence speed of our I-AF scheme for various options of [K,N,M]. We can observe that the I-AF algorithm converges faster with a small number of user-pairs (K). Since large amount of the sum-rate can be achieved with relatively small number of iterations, one can save complexity at the expense of graceful degradation in performance.

Fig. 3 exhibits the average sum-rate performance for $[3, 3, 2]$ systems as a function of SNR. As shown in Fig. 3, our AF relay schemes and the iterative Tx beamforming scheme achieve the multiplexing gain of 3 while the naive AF relay beamforming and the iterative AF relay beamforming achieve the multiplexing gain of 2. Since $[3, 3, 2]$ systems have only one nullspace when designing the transmit beamforming, the performance of the iterative Tx beamforming and the conventional ZF scheme is identical. Additionally, we observe that the E-ZF and I-AF schemes offer gain from the relay beamforming. While the gain obtained from the E-ZF is relatively small (about 0.5 dB), the I-AF provides considerable performance gain over the conventional ZF, resulting in more than 3 dB at low SNR regime. The main reason for this difference is that the search space of the E-ZF is limited to $N^2 - K(K-1)$ while that for the I-AF is much wider than this due to the multiple runs starting from distinct codebook and fast line search using Armijo's rule. Note also that our E-ZF and I-AF scheme obtain more than 10% gain in sum-rate at 30 dB over the naive AF and the iterative AF relay beamforming schemes. We observe similar but improved result in Fig. 4 for system with $[K, N, M] = [6, 6, 4]$. Since there are 2 nullspaces for the iterative Tx beamforming at the transmitter, the iterative Tx beamforming obtains SNR gains over conventional ZF. However, we see that the proposed I-AF provides more than 15% gain in sum-rate at 30 dB over the iterative Tx beamforming.

VI. Conclusions

In this paper, we considered the design of the transmit/receive and relay beamforming schemes when multiple user-pairs are communicating via intermediate relays. By employing the SSA-based transmit and receive beamforming and the improved ZF relay beamforming, we could achieve the multiplexing gain of K and also additional beamforming gain. One of the main assumptions in our work is that there exists a zero-delay error-free feedback link. A related avenue of future work is to study the impact of the mismatch between the channels. Also, extensions to more than two hops is an important direction to be addressed.

REFERENCES

[1] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Select. Areas Commun.,* vol. 25, pp. 379–389, Feb. 2007.

[2] C. Wang, H. Chen, Q. Yin, A. Feng and A.F. Molisch, "Multi-user two-way relay networks with distributed beamforming," *IEEE Trans. Wireless Commun.,* vol.10, no.10, pp.3460-3471, October 2011

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part I: System description," *IEEE Trans. Commun,* vol. 51, pp. 1927-1938, Nov 2003.

[4] A. U. T. Amah, and A. Klein, "Regenerative multi-group multi-way relaying ," *IEEE Trans. Veh. Technol.,* vol. 60, pp. 3017 - 3029, Sept. 2011.

[5] R. S. Ganesan, T. Weber and A. Klein, "Interference alignment in multi-user two way relay networks," *Proc. VTC Conf.,* pp. 1 - 5, May 2011.

[6] K. J. Lee, H. Sung, E. Park and I. Lee, "Joint optimization for one and two-way MIMO AF multiple-relay systems," *IEEE Trans. Wireless Commun.,* vol.9, no.12, pp.3671-3681, December 2010.

[7] N. Lee, J. -B. Lee, and J. Chun, "Degrees of freedom on the MIMO Y channel: signal space alignment for network coding," *IEEE Trans. Inform. Theory,* vol. 56, pp. 3332 - 3342, July 2010.

[8] V. R. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inform. Theory,* vol. 54, pp. 3425-3441, Aug. 2008.

[9] E. Yilmaz, R. Zakhour, D. Gesbert, and R. Knopp, "Multi-pair two-way relay channel with multiple antenna relay station," *Proc. of IEEE ICC,* pp. 1-5, May 2010.

[10] Z. Ding, T. Wang, M. Peng, W. Wang and K.K. Leung, "On the design of network coding for multiple two-way relaying channels," *IEEE Trans. Commun,* vol.10, no.6, pp.1820-1832, June 2011.

[11] S. Xu and Y. Hua, "Optimal design of spatial source-and-relay matrices for a non-regenerative two-way MIMO relay system," *IEEE Trans. Wireless Commun.,* vol.10, no.5, pp.1645-1655, May 2011.

[12] M. S. Bazaraa, H. D. Sherali and C.M. Shetty, *Nonlinear Programming: Thory and Algorithms,* New York: John Wiley & Sons, 3rd edition, 2006

[13] G. H. Goulob and C. F. van Loan, *Matrix Computations,* The Johns Hopkins University Press, Baltimore, 3rd edition, 1996.

TABLE I

E-ZF RELAY BEAMFORMING ALGORITHM

1) Set the conventional ZF beamforming relay matrix as $\mathbf{W}_{ZF,0}$.
2) Find the null vectors of $\mathbf{I}_{i,j}$ in (19) as
$\left[\text{vec}\left(\mathbf{W}_{ZF,1}\right)\text{vec}\left(\mathbf{W}_{ZF,2}\right)\cdots\text{vec}\left(\mathbf{W}_{ZF,N^2-K(K-1)}\right)\right]$
3) Calculate $R_{sum}\left(\mathbf{W}_{ZF,i}\right)$ for $i=0,1,\cdots,N^2-K(K-1)$
4) Select $\mathbf{W}_E$ that satisfies,
$\mathbf{W}_E = \arg\max_{\mathbf{W}_{ZF,i}} R_{sum}\left(\mathbf{W}_{ZF,i}\right)$

TABLE II

I-AF RELAY BEAMFORMING ALGORITHM

Initialization step	R : the number of random initialization of $\mathbf{w}_0$ I_{max} : the maximum number of iterations κ : threshold for terminating iterations
Main Step	**for** I_{pt}=1:R Initialize $\mathbf{w}=\mathbf{w}_0$ as an arbitrary weighting matrix and $k=1$ **while** $k \leq I_{max}$ or $\|R_{sum}(\mathbf{W}_k)-R_{sum}(\mathbf{W}_{k-1})\| < \kappa$ Calculate $\triangledown\mathbf{C}(\mathbf{w}_{k-1})$ **while** Find minimum integer m satisfying (30) **end** Update $\mathbf{w}_k=\mathbf{w}_{k-1}+\delta\cdot\triangledown\mathbf{C}(\mathbf{w}_{k-1})$ **if** $\|R_{sum}(\mathbf{W}_k)-R_{sum}(\mathbf{W}_{k-1})\| < \kappa$ break **else** k=k+1 **end** **end** **end**

TABLE III

COMPUTATIONAL COMPLEXITY (FLOPS) OF THE PROPOSED METHODS, CONVENTIONAL ZF, AND ITERATIVE TX BEAMFORMING (AT EACH CHANNEL REALIZATION)

Operations	$[K, N, M] = [3, 3, 2]$	$[K, N, M] = [6, 6, 4]$
Conventional ZF + SSA	7.38×10^3	1.10×10^5
Iterative Tx beamforming + SSA [10]	7.38×10^3	4.94×10^5
E-ZF + SSA	9.37×10^3	6.42×10^5
I-AF + SSA	1.65×10^5	8.79×10^6

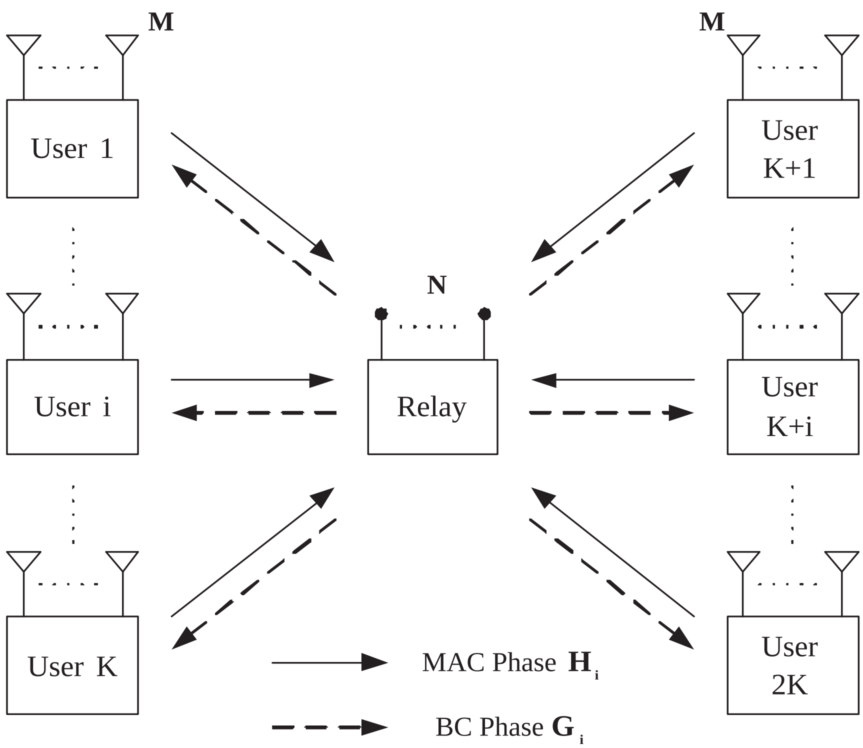

Fig. 1. System model for MIMO multi-pair two-way relay channel.

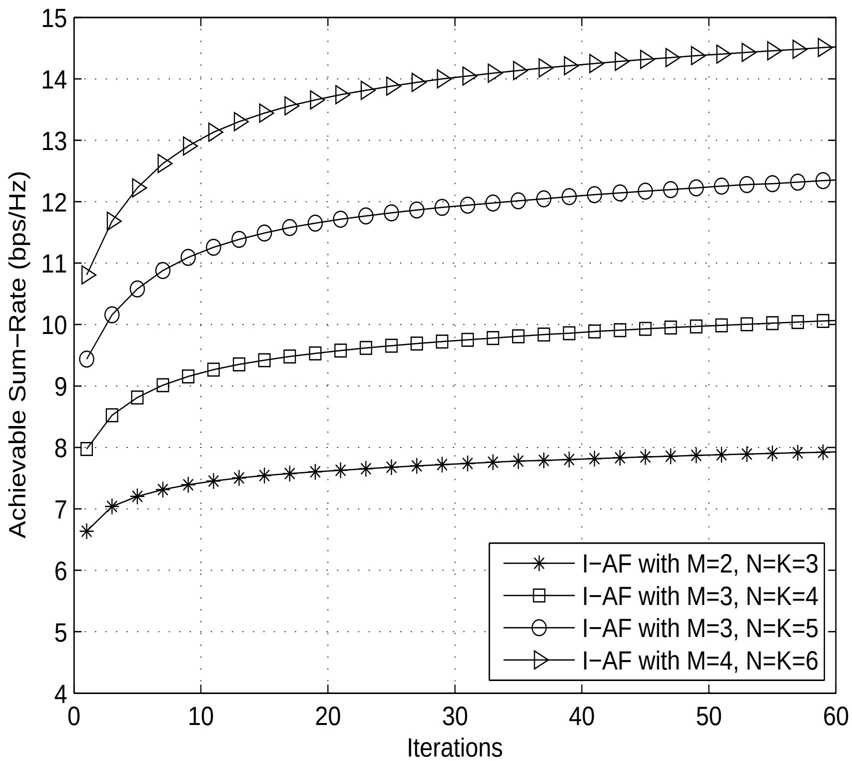

Fig. 2. Sum-rate of the proposed method as a function of the number of iterations.

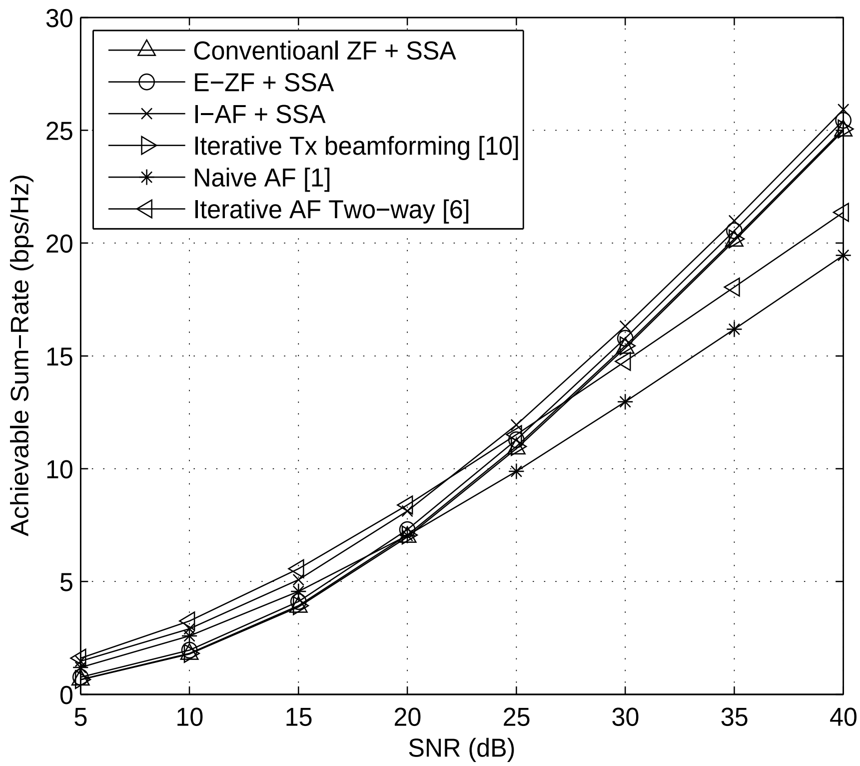

Fig. 3. Sum-Rate vs SNR for 3-pairs with 3 relay antennas and 2 user antennas.

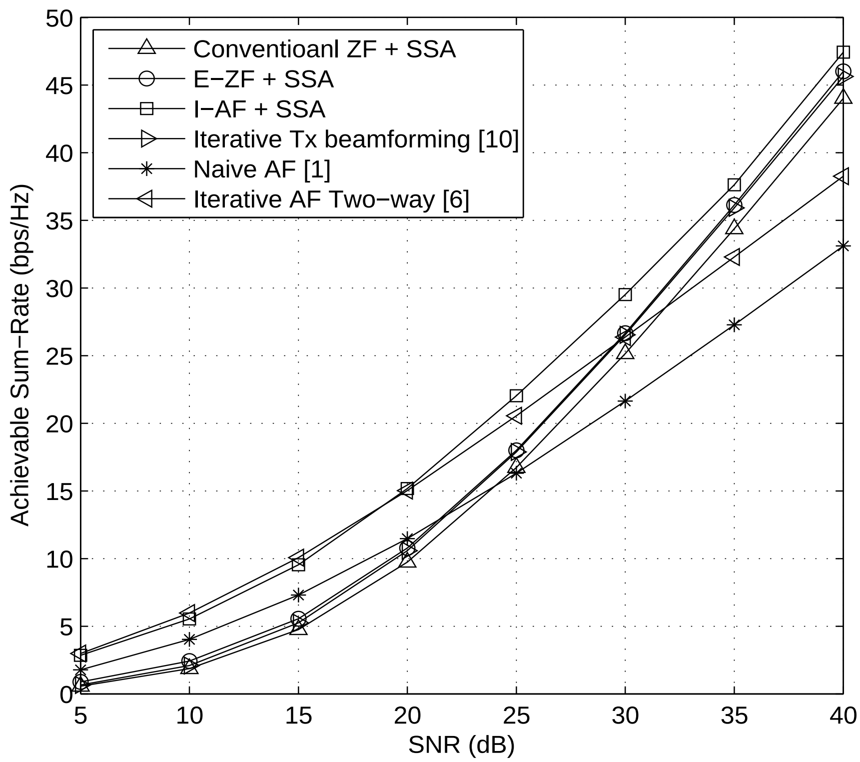

Fig. 4. Sum-Rate vs SNR for 6-pairs with 6 relay antennas and 4 user antennas.