A MIMO Relay with Delayed Feedback Can Improve DoF in K-user MISO Interference Channel with No CSIT

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***Abstract*—This paper studies the impact of a MIMO relay on the K-user MISO interference channel (IC) when the relay has *delayed* feedback (i.e., delayed channel state information (CSI) or delayed channel output feedback) from each receiver, yet each transmitter has no knowledge about CSI. We first develop a new *relay-aided retrospective interference alignment* (r-RIA), and characterize the *optimal* sum-degrees of freedom (DoF) for the K-user MISO IC with a MIMO relay by providing a new matching outer bound. It is shown that the sum-DoF does not decrease even in the absence of direct links between transmitters and receivers, as long as K is sufficiently large. We next show that a MIMO relay can provide a sum-DoF gain in the K-user MISO IC with delayed feedback to the relay in the absence of CSI at transmitter (CSIT) while relaying is shown to be *not* useful with *instantaneous* CSIT from the DoF perspective.**

I. Introduction

In many multi-user wireless networks, interference becomes one of the major hurdles in achieving a significantly high spectral efficiency. It is now well known that interference alignment (IA) [1] is an effective means to greatly alleviate the impact of interference in the interference-limited network. It has been shown that the IA can achieve the optimal degrees of freedom (DoF) in various network settings such as K-user interference channel (IC), X channel, and cellular networks. To achieve a substantial DoF gain that the IA technique promises, accurate instantaneous and global channel state information at transmitters (CSIT) should be available from receivers. When the channel is varying rapidly, the channel feedback is subject to delay, and thus it is hard to achieve the DoF gain due to residual interference effects caused by outdated IA beamforming.

In the seminal work [2], Maddah-Ali and Tse showed that even completely outdated CSIT (i.e., *delayed CSIT*) is helpful in improving the DoF of multiple-input single-output (MISO) broadcast channels (BC). In this work, the role of delayed CSIT is to provide an opportunity for the *sole* transmitter to exploit the overheard interference signals at each receiver as side information, leading to a higher rate than the case with no CSIT in high SNR regime. It is also shown in [2] that the optimal DoF can be achieved with delayed CSIT for the K-user MISO BC with M transmit antennas (if $M \geq K$). In subsequent work, this strategy of using delayed CSIT, referred to as *retrospective interference alignment (RIA)*, has been applied to various wireless channels with distributed transmitters (e.g., K-user IC, X channel, and cellular networks [3]-[8]). The major challenge of the RIA with distributed transmitters is that each transmitter can access only to its own messages, so that the transmitter can only reconstruct the *part* of the side information available at each receiver, even with delayed CSIT. Due to the difficulty of learning the side information, it is in general hard to characterize the optimal DoF when transmitters are distributed except for a few network topologies, such as 2-user X channel and 3-user IC, unlike the broadcast channels where the transmit antennas are co-located.

Recently, there has been extensive prior work to explore the benefits of relays in the interference networks [9]-[11]. It was shown that a relay cannot increase the optimal DoF of the K-user IC [9], yet a relay is useful in facilitating the DoF-optimal scheme of IA in terms of the amount of time/frequency extensions as in [10]. As another beneficial aspect of relay, it turns out that the relay can diminish the amount of channel state information (CSI) feedback towards the transmitters in realizing the optimal DoF. In particular, [11] shows that even in the absence of CSIT (i.e., *blind* transmitters), the optimal DoF with full CSIT can be still attainable since the relay can help to align interference signals with instantaneous and global CSI instead of transmitters.[1] For a practical network design over rapidly time-varying channels, the natural question is whether the relay with completely delayed CSI can still provide a DoF gain for the interference networks without CSIT.

To answer this question, in this work we explore the benefit of the outdated CSI feedback to the relay for the K-user MISO IC with a MIMO relay, when transmitters have no CSI. To tackle typical network connectivity configurations, we consider two distinct scenarios, one of which includes direct links between transmitters and receivers while the other ignores direct links (too weak to be reliably used for data transmission). For both cases, we develop novel relay-aided transmission schemes and fully characterize the *optimal* sum-DoF by providing a new matching outer bounds. We also demonstrate that the relay can provide a significant sum-DoF gain compared to the case where there is no relay under delayed CSIT and no CSIT assumptions.

We briefly summarize notations used in this paper. $\mathbf{A}^T$ and $\mathbf{A}^\dagger$ are the transpose and conjugate transpose of a matrix $\mathbf{A}$, respectively. The operators $\mathsf{rank}(\cdot)$ and $\mathsf{tr}(\cdot)$ represent the

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[1]This is in contrast to the case of no CSIT without the help of the relay where the optimal DoF collapses to one [12].

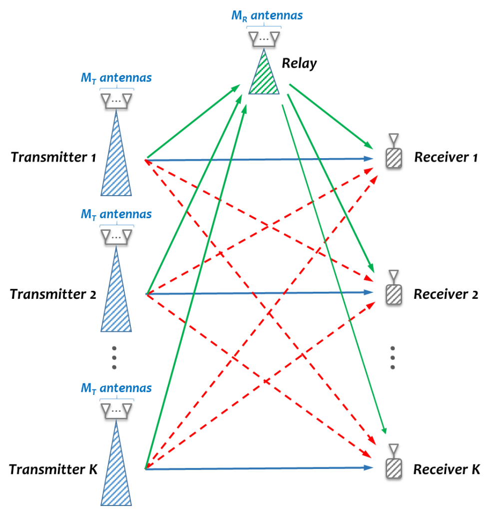

Fig. 1. The diagram of K-user MISO IC with a relay.

rank and the trace of the matrix, respectively. In addition, $\mathbb{E}[\cdot]$ indicates the expectation operator and $\mathbb{C}^{m\times n}$ denotes an $m\times n$ matrix whose components have complex values. A zero-mean circularly symmetric complex Gaussian random variable with variance σ^2 is denoted by $\mathcal{CN}(0,\sigma^2)$.

II. System Model

In this section, we describe a K-user MISO IC equipped with a MIMO relay. As depicted in Fig. 1, each transmitter having M_T multiple antennas sends an independent message $W^{[i]}$ to the receiver having a single antenna by the help of the shared relay having M_R multiple antennas where $M_T, M_R \geq K$. In this work, we assume that the relay employs the two-hop decode-and-forward protocol and half-duplex operation. Since the communication between each transmitter/receiver pair can be regarded as single-cell operation of cellular networks, our system model matches well with a relay-assisted multi-cell cellular network where multiple cells and a common relay share the same frequency band.

The input-output relationship is expressed as

$$\begin{aligned} y^{[i]}(n) &= \mathbf{h}^{[i,i]^T}(n)\mathbf{x}^{[i]}(n) + \sum_{j=1,j\neq i}^{K} \mathbf{h}^{[i,j]^T}(n)\mathbf{x}^{[j]}(n) \quad (1)\\ &\quad +\mathbf{h}^{[i,\mathcal{R}]^T}(n)\mathbf{x}^{[\mathcal{R}]}(n) + z^{[i]}(n), \quad i\in\{1,\cdots,K\}, \\ \mathbf{y}^{[\mathcal{R}]}(n) &= \sum_{j=1}^{K}\mathbf{H}^{[\mathcal{R},j]}(n)\mathbf{x}^{[j]}(n) + \mathbf{z}^{[\mathcal{R}]}(n), \quad (2)\end{aligned}$$

where $y^{[i]}(n)$ and $\mathbf{y}^{[\mathcal{R}]}(n)$ are the received signals at the receiver i and the relay at time slot n, respectively, and $\mathbf{x}^{[i]}(n)$ and $\mathbf{x}^{[\mathcal{R}]}(n)$ are the signals sent by the transmitter i and the relay, respectively. Under the power constraint P, we have $\mathbb{E}\left[\mathrm{tr}\left(\mathbf{x}^{[i]}(n)\mathbf{x}^{[i]^\dagger}(n)\right)\right] \leq P$ at each transmitter and $\mathbb{E}\left[\mathrm{tr}\left(\mathbf{x}^{[\mathcal{R}]}(n)\mathbf{x}^{[\mathcal{R}]^\dagger}(n)\right)\right] \leq P$ at relay, respectively. In addition, $\mathbf{h}^{[i,j]^T}(n) = \left[h_1^{[i,j]}(n), h_2^{[i,j]}(n), \dots, h_{M_T}^{[i,j]}(n)\right] \in \mathbb{C}^{1\times M_T}$, $\mathbf{h}^{[i,\mathcal{R}]^T}(n) = \left[h_1^{[i,\mathcal{R}]}(n), h_2^{[i,\mathcal{R}]}(n), \dots, h_{M_R}^{[i,\mathcal{R}]}(n)\right] \in \mathbb{C}^{1\times M_R}$, and $\mathbf{H}^{[\mathcal{R},j]}(n) \in \mathbb{C}^{M_R\times M_T}$ represent the channel vector from transmitter j to receiver i, the channel vector from the relay to receiver i, and the channel matrix from the transmitter j to the relay, respectively. We further assume that all the entries of $\mathbf{h}^{[i,j]}(n)$, $\mathbf{h}^{[i,\mathcal{R}]}(n)$, and $\mathbf{H}^{[\mathcal{R},j]}(n)$ are drawn from an independent and identically distributed (i.i.d.) continuous distribution. Also, $z^{[i]}(n)$ and $\mathbf{z}^{[\mathcal{R}]}(n)$ denote the additive noise at the receiver i and the relay, respectively, all of whose entries are i.i.d and follow $\mathcal{CN}(0,\sigma^2)$. In our work, we assume that each receiver and the relay have the perfect CSI of its incoming links (i.e., *local* CSI) at each time while no CSI is available at each transmitter. Moreover, we assume that the relay and each receiver have access to the CSI of global links with a one time-slot delay (i.e., *delayed* CSI) except for the incoming links from transmitters.

The sum-DoF characterizing the high SNR behavior of achievable sum-rate is defined as

$$\mathrm{DoF}_{\mathrm{sum}} = \sum_{i=1}^{K} d^{[i]} = \lim_{\mathsf{SNR}\to\infty} \frac{\sum_{i=1}^{K} R^{[i]}(\mathsf{SNR})}{\log(\mathsf{SNR})}, \quad (3)$$

where $d^{[i]} = \lim_{\mathsf{SNR}\to\infty}\frac{R^{[i]}(\mathsf{SNR})}{\log(\mathsf{SNR})}$ and $R^{[i]}(\mathsf{SNR})$ denote the individual DoF and the achievable rate of $W^{[i]}$ for the average power-constraint P, respectively, and $\mathsf{SNR} = \frac{P}{\sigma^2}$.

III. Relay-Aided Retrospective Interference Alignment

In this section, we present the proposed beamforming strategy referred to as *relay-aided retrospective interference alignment (r-RIA)* for the K-user MISO IC with a MIMO relay. The key features of the r-RIA algorithm are 1) *side-information learning* to save an overheard equation involving interference symbols and 2) *relay-aided overheard equation swapping* to exploit the known interference signals based on delayed CSI.

A. Motivating Example ($M_T = M_R = K = 2$)

In this example, we will show that $\frac{4}{3}$ sum-DoF is achievable. Toward this end, each transmitter needs to send two information symbols over three time slots.

1) Phase 1 (Side-Information Learning): This phase consists of two time slots. In the first time slot, transmitter 1 sends two independent symbols $s_1^{[1]}$ and $s_2^{[1]}$ without precoding, i.e., $\mathbf{x}^{[1]}(1) = \left[s_1^{[1]}, s_2^{[1]}\right]^T$ and transmitter 2 keeps a silent. Then, in the absence of the noise[2], receiver 1 obtains a superposition of two desired symbols, i.e., $\mathcal{L}^{[1,1]}(1) = h_1^{[1,1]}(1)s_1^{[1]} + h_2^{[1,1]}(1)s_2^{[1]}$. At the same time, receiver 2 can overhear the interference signal consisting of the undesired symbols, i.e., $\mathcal{L}^{[2,1]}(1) = h_1^{[2,1]}(1)s_1^{[1]} + h_2^{[2,1]}(1)s_2^{[1]}$. In a similar way, for the second time slot, transmitter 2 sends two independent symbols, $s_1^{[2]}$ and $s_2^{[2]}$, while transmitter 1 does not send any of data symbols. Similar to the first time slot, receiver 2 obtains an equation consisting of the two desired symbols, $\mathcal{L}^{[2,2]}(2) = h_1^{[2,2]}(2)s_1^{[2]} + h_2^{[2,2]}(2)s_2^{[2]}$ and

[2]For analytical simplicity, we ignore the noise terms from received signals, which does not affect the sum-DoF when SNR goes to infinity by definition.

receiver 1 overhears the interference signal, i.e., $\mathcal{L}^{[1,2]}(2) = h_1^{[1,2]}(2)s_1^{[2]} + h_2^{[1,2]}(2)s_2^{[2]}$.

It is worth pointing out that if receiver 1 obtains the equation $\mathcal{L}^{[2,1]}(1)$ overheard by receiver 2, then it has enough linear independent equations to decode for its desired two symbols $s_1^{[1]}$ and $s_2^{[1]}$. This is because all the channel coefficients were drawn from a continuous random distribution, so that the full rank condition (i.e., $\text{rank}\left(\left[\mathbf{h}^{[1,1]}(1), \mathbf{h}^{[2,1]}(1)\right]\right) = 2$) is guaranteed almost surely.In a similar way, if receiver 2 observes the equation $\mathcal{L}^{[1,2]}(2)$ overheard by receiver 1, then it has enough linearly independent equations to decode for its desired symbols $s_1^{[2]}$ and $s_2^{[2]}$. In order to show the achievability of $\frac{4}{3}$ sum-DoF, swapping of these two overheard equations should be accomplished in just one time slot with the help of the MIMO relay.

In fact, since the MIMO relay has two antennas ($M_R = 2$), it can decode the messages, $s_1^{[i]}$ and $s_2^{[i]}$, for each time slot $i \in \{1, 2\}$. Thus, at the end of *phase 1*, the relay has a full knowledge of transmit symbols sent from the two transmitters, and hence the channel setting between the relay and the receivers can be readily modeled as a *virtual* MISO BC.

2) Phase 2 (Relay-Aided Overheard Equation Swapping): In the second phase, we use one time slot, $n = 3$. In this phase, all transmitters remain silent and only the relay sends the decoded symbols $s_1^{[i]}$ and $s_2^{[i]}$ to the receivers by applying relay beamforming vectors $\mathbf{v}_1^{[i]} \in \mathbb{C}^{2\times 1}$ and $\mathbf{v}_2^{[i]} \in \mathbb{C}^{2\times 1}$, respectively, i.e.,

$$\mathbf{x}^{[\mathcal{R}]}(3) = \sum_{k=1}^{2} \mathbf{v}_k^{[1]} s_k^{[1]} + \sum_{k=1}^{2} \mathbf{v}_k^{[2]} s_k^{[2]} = \mathbf{V}^{[1]}\mathbf{s}^{[1]} + \mathbf{V}^{[2]}\mathbf{s}^{[2]}, \quad (4)$$

where $\mathbf{V}^{[i]} = \left[\mathbf{v}_1^{[i]}, \mathbf{v}_2^{[i]}\right]$ and $\mathbf{s}^{[i]} = \left[s_1^{[i]}, s_2^{[i]}\right]^T$ for $i \in \{1, 2\}$.

As mentioned, in this phase the relay is aware of the CSI of direct links at $n = 1$ and 2, thus forming the overheard equations $\mathcal{L}^{[2,1]}(1)$ and $\mathcal{L}^{[1,2]}(2)$. An example of $\mathbf{x}^{[\mathcal{R}]}(3)$ is

$$\begin{aligned} \mathbf{x}^{[\mathcal{R}]}(3) &= \begin{bmatrix} \mathcal{L}^{[2,1]}(1) \\ \mathcal{L}^{[1,2]}(2) \end{bmatrix} &(5)\\ &= \underbrace{\begin{bmatrix} \mathbf{h}^{[2,1]}(1)^T \\ \mathbf{0} \end{bmatrix}}_{=\mathbf{V}^{[1]}} \mathbf{s}^{[1]} + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{h}^{[1,2]}(2)^T \end{bmatrix}}_{=\mathbf{V}^{[2]}} \mathbf{s}^{[2]}, \end{aligned}$$

and the corresponding received signals are expressed as

$$\begin{aligned} y^{[1]}(3) &= \mathbf{h}^{[1,\mathcal{R}]}(3)^T \mathbf{x}^{[\mathcal{R}]}(3) &(6)\\ &= h_1^{[1,\mathcal{R}]}(3)\mathcal{L}^{[2,1]}(1) + h_2^{[1,\mathcal{R}]}(3)\mathcal{L}^{[1,2]}(2) + z^{[1]}(3), \\ y^{[2]}(3) &= \mathbf{h}^{[2,\mathcal{R}]}(3)^T \mathbf{x}^{[\mathcal{R}]}(3) &(7)\\ &= h_1^{[2,\mathcal{R}]}(3)\mathcal{L}^{[2,1]}(1) + h_2^{[2,\mathcal{R}]}(3)\mathcal{L}^{[1,2]}(2) + z^{[2]}(3). \end{aligned}$$

Since receiver 1 already has (a noisy version of) $\mathcal{L}^{[1,2]}(2)$, for both desired symbols $s_1^{[1]}$ and $s_2^{[1]}$ can be successfully decoded by combining $y^{[1]}(1)$ and $y^{[1]}(3)$.In a similar manner, receiver 2 can decode the desired symbols by subtracting $y^{[2]}(2)$ from $y^{[2]}(3)$. In summary, since each transmitter is capable of delivering two independent symbols to the intended receiver over three time slots, the achievable sum-DoF is $\frac{4}{3}$.

B. Sum-DoF Analysis for the General Case

In this subsection, we characterize the optimal sum-DoF for the K-user $M_T \times 1$ MISO IC with an M_R-antenna relay. Our main result is formally described in the following theorem.

Theorem 1: For the K-user $M_T \times 1$ MISO IC with a relay having M_R antennas ($M_T, M_R \geq K$), the *optimal* sum-DoF $\text{DoF}_{\text{sum}}^{\text{Relay}} = \frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}$ is achievable if the relay has delayed CSI for the global channel links, yet the transmitters are not able to access any CSI knowledge.

Proof: The achievability of DoF can be shown by leveraging a high-order message transmission strategy in a recursive manner, introduced by Maddah-Ali and Tse [2].[3] In particular, the achievable scheme consists of two phases, a *side-information learning* phase followed by a *relay-aided overheard equations swapping* phase as explained in Section III-A.

The first phase takes K time slots. In this phase, each transmitter alternatively sends $\mathbf{x}^{[n]}(n) = \left[s_1^{[n]}, s_2^{[n]}, \cdots, s_K^{[n]}\right]^T$ through K antennas[4] for $1 \leq n \leq K$ where $s_i^{[n]}$ is the i^{th} order-1 symbol for receiver n. We note that an order-m symbol is defined as a piece of information intended to be delivered to a subset of m receivers. In this sense, the order-1 symbol represents the private message for each receiver.

At the end of phase one, all receivers can observe K linearly independent equations sent from all transmitters. Obviously, among these K linear equations, only one equation is a desired signal and the rest $K-1$ equations are interference signals. For example, receiver 1 can obtain one linear combination of the desired symbols through the first time slot, i.e., $\mathcal{L}^{[1,1]}(1) = \mathbf{h}^{[1,1]^T}(1)\mathbf{s}^{[1]}$. For the time slots from 2 to K, however, it receives the $K-1$ interference signals, i.e., $\mathcal{L}^{[1,n]}(n) = \mathbf{h}^{[1,n]^T}(n)\mathbf{s}^{[n]}$, $n \in \{2, 3, \cdots, K\}$. It must be noted that these interference signals will be exploited as a side information in the second phase to improve the sum-DoF gain of the system.

At this point, the relay can decode all K^2 symbols sent from K transmitter in phase one using the only K out of M_R antennas ($M_R \geq K$), so that it is equivalent to the transmitter in the $K \times 1$ MISO BC under delayed CSIT, if the relay has access to delayed CSI. Similar to [2], the relay can take $(K-j+1)\binom{K}{j}$ common symbols of order-j and generate $j\binom{K}{j+1}$ symbols of order-$(j+1)$ for phase j where $1 \leq j \leq K$.[5] For $j = K$, phase j generates no more symbols, and each phase has $\binom{K}{j}$ time slots, which results in $\text{DoF}_{\text{sum}}^{(j)} = \frac{(K-j+1)\binom{K}{j}}{\binom{K}{j}+j\binom{K}{j+1}/\text{DoF}_{\text{sum}}^{(j+1)}}$ for $1 \leq j < K$ and $\text{DoF}_{\text{sum}}^{(K)} = 1$ where $\text{DoF}_{\text{sum}}^{(q)}$ is the DoF of order-q message for the K-user $K \times 1$ MISO BC. By combining these two achievability results in

[3]The proposed scheme is distinct from RIA [2] in that RIA was developed in the specialized context of K-user MISO BC where no relay is presented.

[4]We need only K antennas to prove the achievability using our achievable scheme, thus the extra $M_T - K$ antennas cannot be exploited to achieve a sum-DoF beyond $\frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}$ due to the tight sum-DoF outer bound [2].

[5]For phase 1 ($j = 1$), the generation step of order-2 symbols at the relay involves the order-1 symbols sent by transmitters in advance, unlike for the succeeding phases where the order-$(j+1)$ symbol generation is based on a function of order-j symbols sent by the relay itself for $j > 1$.

a recursive manner to produce $\mathrm{DoF}_{\mathrm{sum}}^{(1)}$, one can show that the sum-DoF of $\frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}$ is achievable in the relay-aided K-user $M_T \times 1$ MISO IC if the relay has delayed CSI.

The converse argument follows from the fact that a cooperation among any multiple nodes does not degrade the DoF gain of the channel. We thus consider a cooperation scenario in which all K transmitters with no CSI and the relays with delayed CSI fully cooperate with each other by sharing antennas and messages. Under this cooperation setup, we can equivalently convert the original network into a K-user $\widehat{M}_T \times 1$ MISO BC with *partial* delayed CSIT where $\widehat{M}_T = KM_T + M_R > K$. It is worth mentioning that the extra transmit antennas with respect to the number of users (i.e., $\widehat{M}_T - K$) is not helpful in improving the sum-DoF with delayed channel feedback [2]. As a result, the K-user $\widehat{M}_T \times 1$ MISO BC with *global* delayed CSIT has the optimal sum-DoF of $\frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}$ in [2], which is clearly an upper bound for the K-user $M_T \times 1$ MISO IC with a multiple-antenna relay. It happens to coincide with the achievable sum-DoF, thereby establishing the optimality of the proposed r-RIA method. ■

Remark 1: Based on the r-RIA, one can easily show that the K-user $M_T \times N$ MIMO IC with a M_R-antenna relay can achieve $\frac{KN}{1+\frac{1}{2}+\cdots+\frac{1}{K}}$ sum-DoF for arbitrary $M_T, M_R \geq KN$.

Proposition 1: We note that having *delayed Shannon feedback* [3] (delayed channel output feedback (i.e., $y^{[i]}(n-1)$, $\forall i$ at time slot n) as well as delayed CSI) at the relay does not increase the sum-DoF of the K-user MISO IC with delayed CSI at the relay. When all information symbols are decoded successfully at the relay through *phase 1*, delayed output signals can be inferred from delayed knowledge of CSI. Thus, the sum-DoF outer bound for the delayed CSI model derived in *Theorem 1* can be directly applicable to the *delayed Shannon feedback* scenario as well. Indeed, the outer bound is simply achievable by the r-RIA (i.e., discarding delayed channel output feedback). Therefore, the sum-DoF for the K-user $M_T \times 1$ MISO IC with a M_R-antenna relay ($M_T, M_R \geq K$) is given by

$$\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay,FB}} = \frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}, \tag{8}$$

when the relay has delayed output feedback along with delayed CSI.

C. Extension to Layered Two-hop K-user MISO IC with a MIMO Relay

So far, we have considered scenario where the direct links between the transmitters and the receivers are available. We can readily extend the proposed method to the scenario where the direct links between the transmitters and receivers are too weak to be used for data transmission (i.e., $\mathbf{h}^{[i,j]^T} = \mathbf{0}, \forall i, j$ in (1)-(2)), referred to as layered two-hop K-user MISO IC with the MIMO relay. In the absence of direct communication links, the multi-antenna relay is the only transceiver to forward the data symbols between transmitters and receivers so that no overheard side information is available in contrast with *phase 1* of the r-RIA. The following theorem shows that the optimal sum-DoF of K-user MISO IC with MIMO relay can still be achieved even in the absence of direct links between the transmitters and the receivers under a certain assumption.

Theorem 2: For the layered two-hop K-user $M_T \times 1$ MISO IC with a M_R-antenna relay, the optimal sum-DoF is

$$\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay,NoDirect}} \approx \frac{K}{\ln(K)}, \tag{9}$$

provided that K is sufficiently large, where transmitters are not aware of any channel coefficients, yet the relay knows all the completely outdated channel coefficients. In particular, the optimal sum-DoF of the layered two-hop K-user MISO IC (i.e., with *no direct links*) approaches that of the K-user MISO IC with MIMO relay as long as K is large enough.

Proof: In what follows, we first provide some insights on how to achieve the optimal sum-DoF scaling of $\frac{K}{\ln(K)}$. The achievable scheme over the layered two-hop K-user MISO IC proceeds in two phases, a multiple access channel (MAC) phase followed by a RIA phase. During the MAC phase, each transmitter sends intended data symbols to the relay, and the relay jointly detects all the data symbols. During the subsequent RIA phase, the relay with M_R antennas serves as a *virtual* transmitter in the conventional RIA [2] for K-user $M_R \times 1$ MISO BC. Since the extra transmit antennas with respect to the number of users do not provide any gain on DoF, we ignore the extra antennas in the RIA phase, whereas the entire M_R antennas are fully exploited in the MAC phase.

Clearly, the objective of the MAC phase is to deliver K data symbols for each user (i.e., K^2 symbols of order-1 in total) to the relay with no CSIT. To this end, we need at least $\frac{K^2}{\min(M_R, KM_T)}$ time slots due to the cut-set DoF bound. Besides, the RIA phase takes $\frac{K^2}{\mathrm{DoF}_{\mathrm{sum}}^{(1)}}$ time slots to deliver incoming K^2 symbols of order-1 to designated receivers with delayed channel feedback. Therefore, the achievable sum-DoF can be expressed as

$$\begin{aligned}\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay,NoDirect}} &= \frac{K^2}{\frac{K^2}{\min(M_R, KM_T)} + \frac{K^2}{\mathrm{DoF}_{\mathrm{sum}}^{(1)}}} \\ &\overset{(a)}{=} \frac{K}{\frac{K}{\min(M_R, KM_T)} + 1 + \frac{1}{2} + \cdots + \frac{1}{K}},\end{aligned} \tag{10}$$

where (a) follows from the fact that $\frac{K}{\mathrm{DoF}_{\mathrm{sum}}^{(1)}} = 1 + \frac{1}{2} + \cdots + \frac{1}{K}$.

For sufficiently large K, the achievable sum-DoF of the layered two-hop K-user MISO IC approaches the optimal sum-DoF of K-user MISO IC with MIMO relay (i.e., $\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay}}$) where the direct links are actively taken into account for data transmission, i.e.,

$$\begin{aligned}\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay,NoDirect}} &\overset{(a)}{\approx} \frac{K}{\frac{K}{\min(M_R, KM_T)} + \ln(K)} \\ &\overset{(b)}{\geq} \frac{K}{1+\ln(K)} \approx \frac{K}{\ln(K)} \overset{(c)}{\approx} \mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay}},\end{aligned} \tag{11}$$

where (a) and (c) are obtained from the asymptotic approximation of harmonic numbers, i.e., $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k} \approx \lim_{n\to\infty} \ln(n)$, and (b) follows from the following inequality

$$\frac{K}{\min(M_R, KM_T)} = \max\left(\frac{K}{M_R}, \frac{1}{M_T}\right) \leq 1, \ \forall M_T, M_R \geq K.$$

Note that the outer bound of sum-DoF for the K-user MISO IC with MIMO relay is also an outer bound of the layered two-hop K-user MIMO IC, which can be obtained through a full cooperation approach among all transmitters and the relay (see the proof of *Theorem 1*). The converse bound exactly coincides with the achievable sum-DoF when K is sufficiently large, thus we can prove that the optimal sum-DoF is not affected by the existence of direct links on the two-hop K-user MISO IC by comparing $\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay,NoDirect}}$ and $\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay}}$. ■

IV. Discussion: Sum-DoF Gain Analysis from MIMO relay with Delayed CSI

In this section, we compare sum-DoF results according to the existence of relay. From this comparison, we can observe the benefit of exploiting the MIMO relay with delayed channel feedback in the context of K-user MISO IC with no CSIT.

Theorem 3: A MIMO relay with M_R antennas can improve the optimal DoF of K-user $M_T \times 1$ MISO IC with no CSIT when delayed CSI is available at the relay, i.e.,

$$\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay}} > \mathrm{DoF}_{\mathrm{sum}}^{\mathrm{NoRelay}}, \tag{12}$$

where $\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{NoRelay}}$ is the optimal DoF for the MISO IC without relay and $M_T, M_R \geq K \geq 2$.

Proof: From the following *Lemma 1*, we can show that

$$\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay}} = \frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}} \overset{(a)}{>} 1 \overset{(b)}{\geq} \mathrm{DoF}_{\mathrm{sum}}^{\mathrm{NoRelay}}, \tag{13}$$

where the *strict* inequality in (a) holds when $K > 1$ and (b) follows from *Lemma 1*. ■

Lemma 1: For the K-user $M_T \times 1$ MISO IC with no CSIT, the optimal DoF for the case $M_T \geq K$ is bounded by

$$\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{NoRelay}} = \sum_{i=1}^{K} d^{[i]} \leq 1. \tag{14}$$

Proof: The outer bound of the sum-DoF for the K-user $M_T \times 1$ MISO IC with no CSIT can be found by the outer bound to the DoF region of the K-user $M \times N$ MIMO IC with no CSIT presented in [12]. That is,

$$0 \leq d^{[i]} \leq \min(M,N), \quad \sum_{i=1}^{K} \frac{d^{[i]}}{\min(MK,N)} \leq 1. \tag{15}$$

By combining the above two inequalities and realizing that $M \geq N = 1$, we can yield the outer bound result in (14). ■

To shed further light on the impact of the relay with delayed CSI, it is instructive to compare the derived sum-DoF of $\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay}}$ with the existing sum-DoF results in [5]-[7] under the assumption of delayed CSIT without the help of the relay.

Remark 2: For the K-user MISO IC with delayed CSIT, there is no prior work addressing the optimal sum-DoF due to the lack of tight upper bounds. However, it turns out that 2-phase RIA schemes can achieve the sum-DoF of $\frac{K^2}{K^2-K+1}$ in [5] and $\frac{2K}{K+1}$ in [6], which are strictly greater than that of no CSIT case. Subsequently, a higher sum-DoF can be achieved by K-phase RIA [7], and it is bounded above by a constant, $64/15 \approx 4.267$, which does not scale with the number of users K. To the best of our knowledge, there is no transmission scheme achieving higher sum-DoF than K-phase RIA [7] in the K-user MISO IC with delayed CSIT. In this sense, we conjecture that exploiting the MIMO relay with delayed CSI is much more beneficial compared to conveying delayed channel feedback to all transmitters on the K-user MISO IC in terms of DoF scaling for large values of K although no optimality argument can be made due to the loose upper bound for the delayed CSIT setting in the literature:

$$\begin{aligned}\lim_{K\to\infty} \mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Relay}} &\approx \lim_{K\to\infty} \frac{K}{\ln(K)} \\ &\gg \frac{64}{15} = \lim_{K\to\infty} \mathrm{DoF}_{\mathrm{sum}}^{\mathrm{NoRelay,D\text{-}CSIT}},\end{aligned} \tag{16}$$

where $\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{NoRelay,D\text{-}CSIT}}$ is the best known achievable sum-DoF by K-phase RIA [7].

V. Conclusion

In this paper, we have investigated the impact of a MIMO relay having M_R antennas for the K-user $M_T \times 1$ MISO IC, when delayed channel feedback is available at the relay. We have shown that with delayed channel feedback, the K-user MISO IC has $\frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}$ sum-DoF, which turns out to be optimal. The achievability was shown using the r-RIA where the relay can fully exploit the side information based on delayed CSI. We also showed that the sum-DoF scaling is maintained even in the absence of the direct links. Remarkably, we found that the relay can provide a significant sum-DoF gain in the K-user MISO IC with delayed CSIT as well as no CSIT.

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