Joint Modulation Classification and Detection using Sphere Decoding

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***Abstract*—In this letter, we propose a simple yet effective modulation classification method for maximum likelihood multi-user detection. Our method is a modification of generalized likelihood ratio test (GLRT) that approximates the optimal classifier in the Bayesian sense. We show that the proposed method can be implemented by modifying the sphere decoding algorithm to support multi-modulation. Simulation results in multi-user detection in high-speed downlink packet access (HSDPA) system show that the proposed method offers considerable performance gain over conventional RAKE and MMSE algorithms.**

***Index Terms*—Modulation classification, sphere decoding, multi-user detection, generalized likelihood ratio test (GLRT).**

I. Introduction and Motivation

Accurate modeling of system, whether it is given inherently or needs to be estimated explicitly, is critical for maximum likelihood (ML) detection of communication systems. In particular, recent advance in ML detection technique so called *Sphere Decoding* (SD) algorithm enables the receiver to achieve the minimum probability of error performance with controllable complexity [1]–[3]. Besides the accurate estimate for channel matrix, modulation set information is required for the ML detection. While this information is typically available for single-user communication system, such is not true for multi-user system described by

$$\mathbf{y} \,=\, \mathbf{T}\mathbf{s} + \mathbf{v} = \mathbf{T}_0\mathbf{s}_0 + \sum_{i>0} \mathbf{T}_i\mathbf{s}_i + \mathbf{v} \tag{1}$$

where $\mathbf{s}_0$ and $\mathbf{s}_i$ are the symbol vectors of the user of interest and other users, $\mathbf{T}_0$ and $\mathbf{T}_i$ are the associated channel matrices, and $\mathbf{v}$ is the additive noise. This setup is typical in modeling multi-user interference in the base station receiver and intercell interference in the mobile station receiver. In general, the modulation set of the user of interest is assumed to be known while those of others are unknown to the receiver.

The received signal SNR depends on how to treat the interference $\sum_{i>0} \mathbf{T}_i\mathbf{s}_i$. Standard single user detector (SUD) such as linear MMSE equalizer or matched filter [5], [6] cannot exploit the full potential of the received signal since they treat the interference as an additional noise. As a result, the effect of the SUD to mitigate the interference is essentially marginal. The system model of the SUD is

$$\mathbf{y} \,=\, \mathbf{T}_0\mathbf{s}_0 + \mathbf{v}' \tag{2}$$

where $\mathbf{v}' \,=\, \sum_{i>0} \mathbf{T}_i\mathbf{s}_i \,+\, \mathbf{v}$ and hence the receiver SNR becomes

$$\text{SNR}_{\text{sud}} = \frac{E|\mathbf{T}_0\mathbf{s}_0|^2}{\sum_{i>0} E|\mathbf{T}_i\mathbf{s}_i|^2 + \sigma_v^2} \tag{3}$$

where $\sigma_v^2 = E|\mathbf{v}|^2$. Whereas, the receiver SNR of the multi-user detector (MUD) is

$$\text{SNR}_{\text{mud}} = \frac{E|\mathbf{T}\mathbf{s}|^2}{\sigma_v^2}. \tag{4}$$

Since SNR_{mud} is in general much higher than SNR_{sud}, we may expect substantial performance gain by using the MUD if the modulation information of other users are accurately judged by the receiver.

In this letter, we propose a simple modulation classification method that enables the ML detection of multi-user communication system. Contrary to the previous studies focusing on either the modulation classification [7], [8] or the MUD with uniform modulation [4], our method pursues a joint classification and ML detection. In essence, our classification method is a simple modification of generalized likelihood ratio test (GLRT) among multiple symbol set hypotheses. It is shown that this method is an approximation of optimal classifier in the Bayesian sense requiring moderate number of observations and computational complexity. Henceforth, while significantly reducing the computational burden of Bayesian approach, proposed method achieves excellent classification performance, which directly brings considerable performance gain of the multi-user communication system. The key ingredient of our method is the SD algorithm supporting multi-modulation format. In fact, the ML cost function, which is the sufficient statistic of the classification, is a by-product of SD algorithm so that both the classification as well as the ML detection is achieved by a simple modification of the SD algorithm.

The rest of this letter is organized as follows. In section II, we present the proposed modulation classification method based on the modified GLRT. The simulation results are provided in section III and we conclude in section IV.

II. Modulation Classification

A. Problem Setup

For the modulation classification, we need to perform the hypothesis testing among all possible modulation formats the

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transmitter uses. In this section, we focus on the binary hypothesis testing where the decision is made in favor of a modulation scheme over the other (e.g., QPSK and 16-QAM schemes in HSDPA scenario). We mention that the extension to general M-ary ($M = I^{N-1}$ where N is the number of other users and I is the number of possible modulation sets that each user's signal can take) hypothesis testing is straightforward. In this model, $\mathbf{s}_1$ is the m_1-dimensional unknown user vector either from $\Lambda_0 = \mathcal{X}_0^{m_1}$ or $\Lambda_1 = \mathcal{X}_1^{m_1}$, where $\mathcal{X}_i$ $(i = 0, 1)$ is the set of modulation symbol. For example, $\mathcal{X}_0 = \{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}$ for 2-PAM and $\mathcal{X}_1 = \{-\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\}$ for 4-PAM system[1]. Additionally, we assume that m_0-dimensional known signal $\mathbf{s}_0$ is from $\tilde{\Lambda} = \tilde{\mathcal{X}}^{m_0}$. In this setup, the classification problem is given by

$$\begin{cases} \mathcal{H}_0: & \mathbf{y} \sim P_r(\mathbf{y}\,;\, \mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_0) \\ \mathcal{H}_1: & \mathbf{y} \sim P_r(\mathbf{y}\,;\, \mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_1). \end{cases} \tag{5}$$

Since Λ_i is an unknown parameter, this test is categorized as a composite hypothesis testing. Two well-known methods, viz. Bayesian approach and generalized likelihood ratio test (GLRT), may be employed to generate a decision statistic of the composite hypothesis testing [6].

B. Bayesian vs. GLRT based Modulation Classification

In Bayesian approach, a prior distribution on $\mathbf{s}_0$ and $\mathbf{s}_1$ is provided. Then the probabilities for each hypothesis in (5) become

$$P_r(\mathbf{y}\,;\,\Lambda_i) = \sum_{\mathbf{s}_0 \in \tilde{\Lambda},\, \mathbf{s}_1 \in \Lambda_i} P_r(\mathbf{y}\,|\,\mathbf{s}_0, \mathbf{s}_1) P_r(\mathbf{s}_0, \mathbf{s}_1),\ i = 0, 1. \tag{6}$$

Under the uniform assumption on priors ($P_r(\mathbf{s}_1) = \frac{1}{|\mathcal{X}_i|^{m_1}}$) and using the statistically independence between $\mathbf{s}_0$ and $\mathbf{s}_1$, the likelihood ratio of Bayesian approach becomes

$$\frac{\sum_{\mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_1} P_r(\mathbf{y}\,|\,\mathbf{s}_0, \mathbf{s}_1) P_r(\mathbf{s}_1)}{\sum_{\mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_0} P_r(\mathbf{y}\,|\,\mathbf{s}_0, \mathbf{s}_1) P_r(\mathbf{s}_1)} = \frac{\sum_{\mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_1} P_r(\mathbf{y}\,|\,\mathbf{s}_0, \mathbf{s}_1) |\mathcal{X}_0|^{m_1}}{\sum_{\mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_0} P_r(\mathbf{y}\,|\,\mathbf{s}_0, \mathbf{s}_1) |\mathcal{X}_1|^{m_1}}. \tag{7}$$

Since $P_r(\mathbf{y}\,|\,\mathbf{s}_1, \mathbf{s}_0) = \frac{1}{(2\pi\sigma^2)^{m/2}} \exp\left(-\frac{1}{2\sigma^2} ||\,\mathbf{y} - \mathbf{T}\mathbf{s}\,||^2\right)$, we obtain the log-likelihood ratio (LLR) by taking a logarithm as

$$\text{LLR} = m_1 \log \frac{|\mathcal{X}_0|}{|\mathcal{X}_1|} + \log\left(\frac{\sum_{\mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_1} \exp\left(-\frac{1}{2\sigma^2} ||\,\mathbf{y} - \mathbf{T}\mathbf{s}\,||^2\right)}{\sum_{\mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_0} \exp\left(-\frac{1}{2\sigma^2} ||\,\mathbf{y} - \mathbf{T}\mathbf{s}\,||^2\right)}\right). \tag{8}$$

Since the computational complexity of (8) is proportional to $|\tilde{\mathcal{X}}|^{m_0}(|\mathcal{X}_0|^{m_1} + |\mathcal{X}_1|^{m_1})$, this approach is computationally infeasible in most multi-user detection scenarios. In contrast to this brute-force natured Bayesian method, GLRT relies only on the best $\mathbf{s}$ for each hypothesis [6]. Specifically, GLRT approach computes $\mathbf{s}$ for each hypothesis by the ML detection and then use it as a true parameter in the likelihood ratio test (LRT). The likelihood ratio of the GLRT is

$$\frac{\max_{\mathbf{s}_0 \in \tilde{\Lambda},\, \mathbf{s}_1 \in \Lambda_1} P_r(\mathbf{y}|\,\mathbf{s}_0, \mathbf{s}_1)}{\max_{\mathbf{s}_0 \in \tilde{\Lambda},\, \mathbf{s}_1 \in \Lambda_0} P_r(\mathbf{y}|\,\mathbf{s}_0, \mathbf{s}_1).} \tag{9}$$

(9) can be concisely rewritten as

$$\frac{P_r(\mathbf{y}|\,\hat{\mathbf{s}}(\Lambda_1))}{P_r(\mathbf{y}|\,\hat{\mathbf{s}}(\Lambda_0))} \tag{10}$$

where $\hat{\mathbf{s}}(\Lambda_i) = \arg\min_{\mathbf{s}_0 \in \tilde{\Lambda}, \mathbf{s}_1 \in \Lambda_i} ||\,\mathbf{y} - \mathbf{T}\mathbf{s}\,||^2$. Then the LLR of the GLRT becomes

$$\text{LLR} = -\frac{1}{2\sigma^2} ||\,\mathbf{y} - \mathbf{T}\hat{\mathbf{s}}(\Lambda_1)\,||^2 + \frac{1}{2\sigma^2} ||\,\mathbf{y} - \mathbf{T}\hat{\mathbf{s}}(\Lambda_0)\,||^2. \tag{11}$$

Note that (11) is nothing but a scaled version of the ML cost difference and hence the classification rule is to compare sufficient statistics $J(\hat{\mathbf{s}}(\Lambda_1)) = ||\,\mathbf{y} - \mathbf{T}\hat{\mathbf{s}}(\Lambda_1)\,||^2$ and $J(\hat{\mathbf{s}}(\Lambda_0)) = ||\,\mathbf{y} - \mathbf{T}\hat{\mathbf{s}}(\Lambda_0)\,||^2$. That is, we choose $\mathcal{H}_1$ if $J(\hat{\mathbf{s}}(\Lambda_0)) > J(\hat{\mathbf{s}}(\Lambda_1))$ and $\mathcal{H}_0$ otherwise. Extending the GLRT detection for multiple observation $\underline{\mathbf{y}} = [\mathbf{y}_1\, \mathbf{y}_2\, \cdots\, \mathbf{y}_N]^T$, one can easily show that the LLR is $-\frac{1}{2\sigma^2}\sum_{k=1}^N J(\hat{\mathbf{s}}_k(\Lambda_1)) + \frac{1}{2\sigma^2}\sum_{k=1}^N J(\hat{\mathbf{s}}_k(\Lambda_0))$. In short, the decision is made by observing the difference between the sum of ML cost function for each hypothesis.

C. Modified GLRT

Although the LLR of GLRT in (11) is much easier to implement than the Bayesian approach in (8), due to the difference in the number of constellation points, high-order modulation (e.g., 16-QAM) is favorably chosen over low-order modulation (e.g., QPSK), in particular for the low SNR regime. As a simple example, when the noise magnitude is very large, $J(\mathbf{s}_k(\Lambda_1)) < J(\mathbf{s}_k(\Lambda_0))$ and thus 16-QAM is always detected. Clearly, the situation will get worse when the comparison is between QPSK and 64-QAM. In fact, we can interpret that (11) is equivalent to the log term of (8) by the Jacobian-logarithm approximation. Since the bias term $m_1 \log \frac{|\mathcal{X}_0|}{|\mathcal{X}_1|}$ is ignored in the GLRT, we need to incorporate this to get an approximation of the Bayesian test. The modified classification rule is to choose $\mathcal{H}_1$ if

$$\sum_k J(\hat{\mathbf{s}}_k(\Lambda_0)) + 2\sigma^2 m_1 \log|\mathcal{X}_0| \geq \sum_k J(\hat{\mathbf{s}}_k(\Lambda_1)) + 2\sigma^2 m_1 \log|\mathcal{X}_1| \tag{12}$$

and choose $\mathcal{H}_0$ otherwise. Since the ML cost function $J(\hat{\mathbf{s}}_k(\Lambda_i))$ can be obtained as a by-product of the ML detection, $\hat{\mathbf{s}}_k(\Lambda_i)$ can be computed by a simple modification of the SD algorithm.

Recall that the conventional SD algorithm solves the ML solution given by [3]

$$\min_{\mathbf{s} \in \mathcal{X}^m} ||\,\mathbf{y} - \mathbf{T}\mathbf{s}\,||^2, \tag{13}$$

Whereas, ML detection problem computing $\hat{\mathbf{s}}(\Lambda_i)$ in our method should solve

$$\min_{\mathbf{s}_0 \in \tilde{\mathcal{X}}^{m_0}, \mathbf{s}_1 \in \mathcal{X}_i^{m_1}} ||\,\mathbf{y} - \mathbf{T}\mathbf{s}\,||^2. \tag{14}$$

[1] Since the SD algorithm is based on the real-valued system model [3], we separated real and imaginary parts of QPSK and 16-QAM modulations in this example.

This problem can be easily solved by the search set adjustment of the SD algorithm. Expressing the system as $\mathbf{y} = \mathbf{T}[\mathbf{s}_0^T \ \ \mathbf{s}_1^T]^T + \mathbf{v}$, the real-valued signal model becomes

$$\mathbf{y}_{re} = \mathbf{T}_{re}\mathbf{s}_{re} + \mathbf{v}_{re} \tag{15}$$

By applying the QR decomposition of $\mathbf{T}_{re}$ ($\mathbf{T}_{re} = \mathbf{QR}$), the ML problem in (14) becomes triangular structured $\min_{\mathbf{s}_0 \in \tilde{\mathcal{X}}^{m_0}, \mathbf{s}_1 \in \mathcal{X}_i^{m_1}} ||\,\mathbf{p}(\mathbf{s}_0, \mathbf{s}_1)\,||^2$ where $\mathbf{p}(\mathbf{s}_0, \mathbf{s}_1) = \mathbf{Q}^T\mathbf{y}_{re} - \mathbf{R}\mathbf{s}_{re} = \bar{\mathbf{y}} - \mathbf{R}\bar{\mathbf{s}}$. From the sphere constraint $\sum_{i=k}^{m} |p_i(\mathbf{s}_0, \mathbf{s}_1)|^2 \leq d_0$ at layer $m - k + 1$ where $p_i(\mathbf{s}_0, \mathbf{s}_1)$ is the i-th row of $\mathbf{p}(\mathbf{s}_0, \mathbf{s}_1)$, one can find the basic searching set as

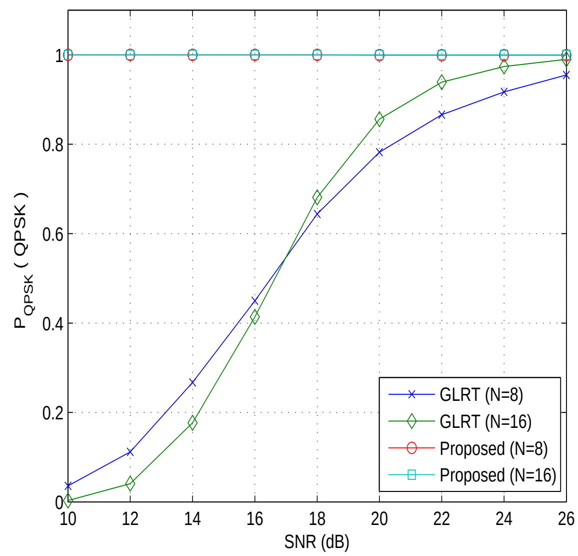
$$\mathcal{S}^{(m-k+1)} = \{\bar{s}_k \in \mathcal{Z} \mid \frac{\bar{y}_k - \sum_{j=k+1}^{m} r_{k,j}\bar{s}_j - \sqrt{d_0}}{r_{k,k}} \leq \bar{s}_k \leq \frac{\bar{y}_k - \sum_{j=k+1}^{m} r_{k,j}\bar{s}_j + \sqrt{d_0}}{r_{k,k}}\}. \tag{16}$$

Due to the modulation format change, the searching set of the layer k should be adjusted to either $\mathcal{S}^{(k)} \cap \mathcal{X}_i$ or $\mathcal{S}^{(k)} \cap \tilde{\mathcal{X}}$ depending on the modulation format of $\bar{s}_k$.

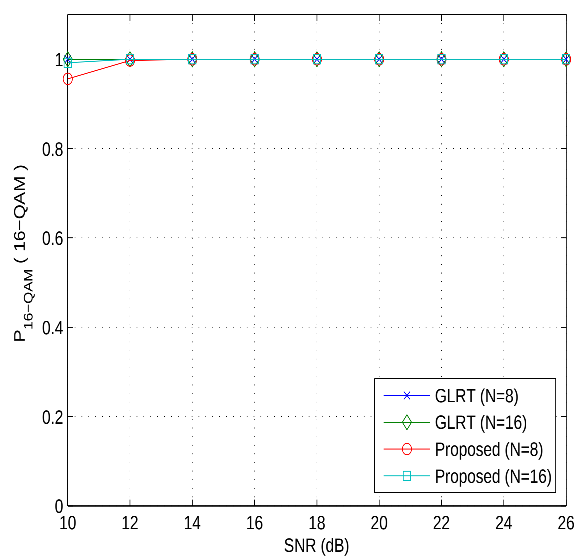
III. Simulations Results

In this section, we demonstrate the effectiveness of the proposed algorithm in HSDPA environment of UMTS system [11], where the downlink reception is coming from two cells (primary and secondary). In order to model a mobile in the cell-edge, the power of primary cell containing a desired user is set 3 dB higher than that of interfering cell (SIR = 3 dB). In addition, each cell has four distinct multipaths and the relative powers for each path are 0, -6, -9, and -12 dB, respectively ($h[n] = 0.833\delta[n] + 0.418j\delta[n-1] - 0.296\delta[n-2] + 0.209\delta[n-3]$). Both desired user in a primary cell as well as interferer in a secondary cell employ three data channels (HS-PDSCH) ($m_0 = m_1 = 3$). The modulation scheme of the desired user in the primary cell is fixed with QPSK while that of the interferer in the secondary cell is randomly flipping between QPSK and 16-QAM per 2 ms transmission time interval (TTI). For the modulation classification, we test the GLRT as well as the modified GLRT method with two distinct symbol accumulations ($N = 8$ and 16). Noting that 480 symbols are transmitted per TTI, the corresponding classification period is 1.7% and 3.3%, respectively. As a system performance metric, we employ symbol error rate (SER) of desired data channels in the primary cell. In obtaining the SER curve, we simulate 1000 TTI (2 sec period) for each SNR point.

Fig. 1(a) and (b) provide the classfication performance ($P_{\text{QPSK}}(\text{QPSK})$ and $P_{\text{16-QAM}}(\text{16-QAM})$) for QPSK and 16-QAM transmission. Note that $P_{\text{QPSK}}(\text{QPSK})$ denotes the probability that QPSK modulation is decided when unknown symbol format is QPSK ($P_{\text{16-QAM}}(\text{16-QAM})$ is interpreted in a same way). Since the GLRT provides favorable decision over 16-QAM (refer Sec. II-C), the performance of the GLRT when QPSK is transmitted is poor in low SNR regime while the modified GLRT detection achieves perfect classification performance in the entire simulation range. Whereas, all classification methods we tested show the excellent classification performance on 16-QAM transmission except for the modified GLRT at the low SNR regime. Even in this case, by slightly increasing the number of observations, we could achieve near perfect classification.



(a)



(b)

Fig. 1. Classification performance for SNR variation: (a) QPSK. (b) 16-QAM transmission.

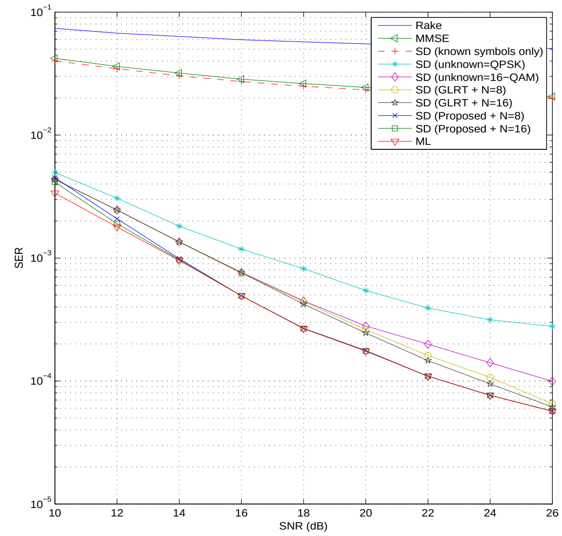
As an eventual measure for observing overall performance, we next examined the SER performance employing the proposed classification method. For comparison, we also included the ML detector (SD with perfect modulation knowledge) as well as conventional receiver algorithms including RAKE, MMSE, and SD with known channel only. In general, the performance of conventional receivers is poor due to the existence of interferences in the secondary cell as shown in Fig. 2(a). Whereas, SD families show noticeably better performance over conventional methods. In particular, the performance curves of the proposed method and the ML receiver lie on top of each other in mid and high SNR regime. Even in low SNR regime, maximum performance gap from the ML detector is only about 0.3 dB.

In order to quantify the complexity of proposed method, we also checked the average number of nodes visited for all SD families. Note that during the classification period, the complexity of SD algorithms enforcing the modulation of unknown user to QPSK (say QPSK-SD) and 16-QAM (say 16QAM-SD) is both considered in the proposed methods to account for the classification complexity. As shown in Fig. 2(b), we observe that QPSK-SD and 16QAM-SD show the best and worst complexity and the proposed method and GLRT based SD algorithm lie between them. Due to the fact that 16-QAM is favorably chosen over QPSK, the complexity of GLRT based SD method is close to the 16QAM-SD in low and mid SNR regime. Whereas, the complexity of proposed SD method is very close to the ML detector in mid and high SNR regime mainly because 1) the classification performance in this regime is nearly perfect (see Fig. 1) and 2) the classification period is really small so that the impact of the classification process on overall complexity is quite small. Note, since single or at most a few users are assigned in a cell for a given TTI, complexity overhead of the modulation classification will be moderate in most operating scenario of HSDPA.

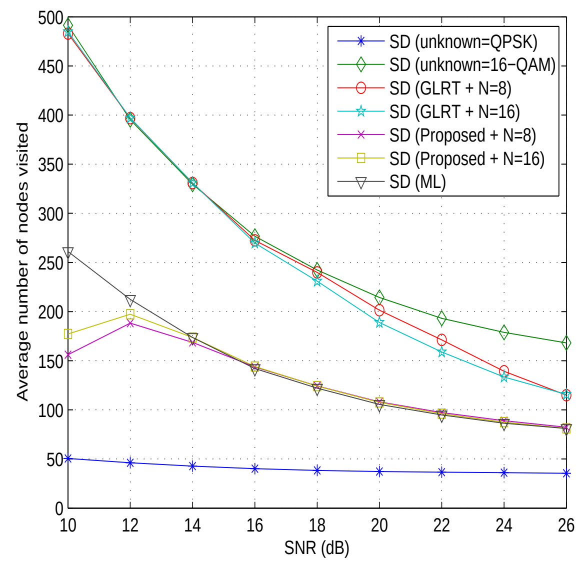
IV. Conclusions

In this letter, we presented a modulation classification method for maximum-likelihood multi-user detection. We showed that simple modification of the SD algorithm enables the modulation classification as well as the maximum-likelihood multi-user detection. From the simulation on HSDPA system, we observed that the proposed method provides considerable gain over conventional receiver algorithms.

Although this letter demonstrated binary modulation hypotheses testing between QPSK and 16-QAM, the extension to the M-ary (M $>$ 2) problem such as the modulation classification among QPSK, 16-QAM, and 64-QAM scenario can be easily obtained. Even in this case, with the help of advanced SD algorithms [9], [10], implementation complexity can be managed to the reasonable level. Regarding the choice of symbols before the classification operation is done, one might choose $\hat{\mathrm{s}}(\Lambda_1)$ (symbol by 16-QAM modulation hypothesis) at the expense of slight performance loss or store the symbols $\hat{\mathrm{s}}(\Lambda_i)$ in a memory and choose the right one after the classification.



(a)



(b)

Fig. 2. Performance and complexity for SNR variation: (a) SER performance. (b) Complexity.

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