Towards Faster-Than-Nyquist Transmission for Beyond 5G Wireless Communications

Byungju Lee*, Junghyun Kim*, Hyojin Lee*, Byonghyo Shim†, Younsun Kim* and Juho Lee*

*Samsung Research, Samsung Electronics Co., Ltd., Seoul, Korea
†INMC, Department of Electrical and Computer Engineering, Seoul National University, Seoul, Korea
Email: {byungju7.lee, jh0617.kim, hyojin17.lee, younsun, juho95.lee}@samsung.com, bshim@snu.ac.kr

Abstract—Faster-Than-Nyquist (FTN) is a technique that can improve the spectral efficiency of communication systems by making better use of available spectrum resources at the cost of inter-symbol interference (ISI) and inter-carrier interference (ICI). In this paper, we propose a hybrid signaling scheme for a practical application of FTN for MIMO transmission. We propose a new slot structure optimized for the hybrid signaling supporting both FTN signaling and orthogonal frequency division multiplexing (OFDM) signaling. Specifically, in the proposed slot structure, data transmission is based on the FTN signaling and the pilot transmission is based on the OFDM signaling. Numerical results confirm that the proposed signaling scheme has clear benefit over the systems employing only OFDM or FTN signaling.

I. INTRODUCTION

With the advent of 5G wireless communications, we are witnessing unprecedented services such as tactile internet, virtual augmented reality, autonomous driving, factory automations, and high-resolution video streaming, to name just a few [1], [2]. To support wide variety of services and enhanced user experience, we need to push the spectral efficiency of wireless systems to the limit. One viable approach to enhance the spectral efficiency is to transmit the symbol faster than the Nyquist rate. In doing so, symbol rate higher than two times of bandwidth can be achieved at the expense of the orthogonality violation among symbols. Such a transmission scheme, commonly called the faster-than-Nyquist (FTN) signaling [3], has received much attention in recent years as a means to improve the spectral efficiency of wireless systems. It has been shown that 25% faster symbol transmission is achieved by employing deliberately designed pulse shaping filters and binary symbols without virtual loss of performance [3]. It has also been shown in [4] that 42% faster symbol can be achieved by the FTN signaling with square-root raised cosine pulse shaping filter.

Recently, multi-carrier systems employing the FTN signaling have been proposed [5]. Key feature of this scheme over the single carrier counterpart is to improve the bandwidth utilization by reducing the distance between adjacent subcarriers. Although multi-carrier FTN technology has received some research interests, commercial use of this scheme is rare due to the complication caused by the interference-carrier interference (ICI) control. In fact, due to the ICI caused by the overlap of pilot subcarriers, channel estimation quality would be deteriorated, resulting in the severe degradation of the symbol detection and decoding performance. In [6], effect of ICI on the channel estimation has been analyzed when the pilot signals are allocated using the FTN signaling. In [7], an approach to turn off part of subcarriers to ensure the orthogonality of pilot subcarriers has been suggested.

An aim of this paper is to propose an FTN signaling scheme suitable for multiple-input multiple-output (MIMO) transmission. Our work is motivated by the fact that the FTN-based pilot transmission degrades the channel estimation quality severely, in particular for MIMO systems. In order to support more symbols than the conventional OFDM systems can allow while ensuring the accuracy of the channel estimation quality, we propose a hybrid signaling scheme supporting both FTN signaling and OFDM signaling in the transmission time interval (TTI). Key feature of the proposed scheme is to support the data transmission using the FTN signaling and control and pilot signals using the conventional OFDM signaling. Since the pilot signals are transmitted by the OFDM signaling, orthogonality of pilot subcarriers is guaranteed, which ensures that the accurate channel estimation can be achieved even in the MIMO transmission. By controlling the ICI terms of data symbols using the nonlinear receiver techniques (e.g., BCJR [8]), we can achieve the substantial improvement in the spectral efficiency of the data transmission. In our simulations, we show that the proposed hybrid signaling achieves more than 20% gain in the spectral efficiency over the conventional OFDM systems.

II. FTN-BASED PILOT SIGNAL TRANSMISSION

In this section, we present the FTN-based pilot signal transmission and channel estimation and then analyze the effect of ICI on the MIMO transmission. A block diagram of the conventional multi-carrier systems is depicted in Fig. 1(a).

A. Basics on FTN Signal Transmission

Transmit signals of multi-carrier waveform can be expressed as

\[ x(t) = \frac{1}{\sqrt{T}} \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} a_{n,l} e^{j2\pi n\beta \Delta f \left( \frac{t - (l+1)T_{cp}}{T} \right)} \]  

(1)

where \( T \) is the symbol duration, \( a_{n,l} \) is the signal on the \( n \)-th subcarrier of the \( l \)-th symbol and \( T_{cp} \) is the cyclic prefix duration. The subcarrier of the waveform is given by \( \beta \Delta f = \beta \Delta f \) where \( \beta \) is the subcarrier squeezing factor (\( \beta \leq 1 \)). For
example, when $\beta = 0.8$ and $\Delta f = 15$ kHz, then the subcarrier spacing of the FTN waveform is 12 kHz.

In the trivial case ($\beta = 1$), subcarriers of the multicarrier systems are orthogonal to each other [9]. For a given bandwidth $B$ and symbol duration $T$, the number of equally spaced subcarriers should be less than or equal to $\lceil BT \rceil$ to satisfy the orthogonality criterion. In case we violate the orthogonality constraint, we can support more than $\lceil BT \rceil$ subcarriers. For example, if $\beta = 1$, 12 subcarriers with 15 kHz spacing can be supported for the system bandwidth of 180 kHz. Whereas, if $\beta = 0.8$, subcarrier spacing is reduced to 12 kHz and thus 15 subcarriers can be supported for the same bandwidth. Thus, as long as the ICI caused by the FTN signaling is properly controlled, we can obtain a meaningful gain in the spectral efficiency.

B. Channel Estimation in FTN Signaling

The received signal $y_n^{(r)}$ on the $n$-th subcarrier of $r$-th receive antenna is given by

$$y_n^{(r)} = h_n^{(r,t)} p_n^{(t)} + ICI_{data}^{(t)} + ICI_{pilot}^{(t)} + w_n^{(r)}$$

where $h_n^{(r,t)}$ is the channel coefficient from the $t$-th transmit antenna port to the $r$-th receive antenna, $p_n^{(t)}$ is the pilot transmitted from the $t$-th transmit antenna port on the $n$-th subcarrier, and $ICl_{data}^{(t)}$, $ICI_{pilot}^{(t)}$, and $ICI_{pilot}^{(t)}$ are the ICI caused by the adjacent subcarriers. While $ICl_{data}^{(t)}$ is the interference caused by the data subcarriers, $ICI_{pilot}^{(t)}$ and $ICI_{pilot}^{(t)}$ are interferences from the pilots of the same antenna port and different antenna ports, respectively. Specifically, $ICI_{data}^{(t)}$, $ICI_{pilot}^{(t)}$, and $ICI_{pilot}^{(t)}$ can be expressed, respectively, as

$$ICI_{data}^{(t)} = \sum_{k \neq n} c_{n,k} h_k^{(r,t)} p_k^{(t)}$$

$$ICI_{pilot}^{(t)} = \sum_{k \neq n} c_{n,k} h_k^{(r,t)} p_k^{(t)}$$

where $d_k^{(r,t)}$ is the data from the $t$-th transmit antenna port to the $r$-th receive antenna and $c_{n,k}$ is the ICI coefficient between the $n$-th and the $k$-th subcarriers, which is given by

$$c_{n,k} = \left\{\begin{array}{ll}
1, & n = k \\
1 - e^{j2\pi \beta(n-k)}, & n \neq k.
\end{array} \right.$$  \hspace{1cm} (3)

where $Q$ is the number of time samples at a given symbol duration $T$.\footnote{Note that $c_{n,k}$ becomes nonzero in FTN signaling and thus affect ICI on other subcarriers while $c_{n,k}$ becomes zero in OFDM signaling, ensuring no ICI among subcarriers (see Fig. 3a).} Note that $c_{n,k}$ is calculated from the time-domain samples of FTN signal $x[n] = \frac{1}{\sqrt{Q}} \sum_{n=0}^{N-1} a_n e^{j2\pi \frac{\beta(n-k)}{Q}}$. A vector expression of the time-domain FTN signal can be expressed as $x = P a$, where $n$-th element of $a$ is $a_n$ and the $(m,n)$-th element of $P$ is $e^{j2\pi \beta(n-k)}$. Note that the magnitude of $c_{n,k}$ increases when the squeezing factor $\beta$ decreases, meaning that smaller $\beta$ will increase $ICl_{data}$, $ICI_{pilot}^{(t)}$, and $ICI_{pilot}^{(t)}$. In particular, it is very difficult to alleviate $ICl_{data}$ during the channel estimation process since in this case data symbols are treated as a noise.

From this discussion, it is not hard to expect that the frequency division multiplexing (FDM) of the pilot signal and data might not be that effective. Furthermore, controlling the ICI terms in the MIMO transmission is far more difficult since the ICI terms from different antenna ports will be added, degrading the quality of the equalization and channel estimation process severely.

To better illustrate this behavior, we investigate the effect of the FTN-based pilot signal transmission on the channel estimation quality. The received vector from the $t$-th transmit antenna port to the $r$-th receive antenna of $N_p$ pilots can be expressed as

$$y = PCh + w'$

where $P$ is the $N_p \times N_p$ diagonal matrix of the pilot sequence and $w'$ is the sum of noise and ICIs vector obtained from data and other pilots. When the least squares (LS) estimation is applied, the channel estimate can be expressed as

$$\hat{h} = (PC)^{-1}y = h + (PC)^{-1}w'.$$

Since $C$ is a function of $\beta$, its is worth discussing the relationship between the subcarrier squeezing factor $\beta$ and the

\[\text{Authorized licensed use limited to: Seoul National University. Downloaded on October 13,2021 at 02:41:24 UTC from IEEE Xplore. Restrictions apply.}\]
noise power $\| (PC)^{-1} w \|$. Note that the norm of the effective noise $\| (PC)^{-1} w \|$ is upper bounded by $\| (PC)^{-1} w \| = \| C^{-1} P^{-1} w \| = \| C^{-1} w \|$ (since $P^{-1}$ is unitary matrix). When $\beta = 1$, the ICI coefficient matrix $C$ would be the identity matrix and thus the noise power is preserved (i.e., $\| C^{-1} w \| = \| w \|$). On the other hand, when $\beta < 1$, the ICI coefficient matrix $C$ is no more identity matrix and $\| C^{-1} \| = \frac{1}{\lambda_1}$ where $\lambda_1$ is the minimum eigenvalue of $C$. It has been shown in [11] that the minimum eigenvalue $\lambda_1$ decreases as magnitude of correlation coefficient $c_{n,k}$ increases. Since the magnitude of correlation coefficient $c_{n,k}$ increases when $\beta$ decreases (see Fig. 2), the effective noise power will also increase as $\beta$ decreases, degrading the channel estimation performance severely.

C. Discussion on Pilot Signal Transmission

In the previous subsection, we discussed that the channel estimation quality would be deteriorated severely as the subcarrier squeezing factor $\beta$ decreases. One simple way to improve the channel estimation quality is to allocate pilot signals on the FTN symbol such that pilot subcarriers are mutually orthogonal [7]. Note that if the pilot subcarrier difference satisfies the multiple of OFDM subcarrier spacing $\Delta f = \frac{1}{4}$, orthogonality between the $n$-th and $k$-th pilot subcarriers is guaranteed (i.e., $c_{n,k} = 0$). For example, if $\beta = 0.8$, 1st, 6th, and 11th pilot subcarriers are mutually orthogonal. Once the set of mutually orthogonal subcarriers is obtained, the pilot symbol vector $p$ can be mapped as

$$p = \begin{cases} p_i & , \beta (i - 1) \in \mathbb{Z} \\ 0 & , \text{otherwise} \end{cases}$$

Fig. 2: MSE of the channel estimation as a function of the squeezing factor $\beta$ when the FTN-based pilot signal transmission is exploited.

This approach, however, will lead to the inefficient use of time-frequency resources. As mentioned, all subcarriers except for three mutually orthogonal subcarriers should be turned off to prevent ICI on pilot subcarriers.

Some useful observations deduced from this discussion are as follows:

1. FDM of the pilot and data is undesirable since it is difficult to alleviate ICI caused by the data subcarriers in the channel estimation process.
2. FDM of the pilots from the same and different antenna ports is undesirable since it will make equalization for both ICI and MIMO complicated, deteriorating the overall performance.

III. PROPOSED FTN-BASED HYBRID SIGNALING

In this section, we present the proposed hybrid signaling scheme employing the FTN signaling for the data transmission and the OFDM signaling for the ICI-free pilot transmission. Key features of the proposed scheme are as follows:

- A slot, the unit of data scheduling in time domain, is generated by the time division multiplexing (TDM) of OFDM and FTN symbols (see Fig. 3).
- Control and pilot signals are transmitted by the OFDM signaling to guarantee the accurate channel estimation quality. Whereas, data transmission is served by the FTN signaling for better spectral efficiency.

A. Proposed Slot Structure

In 4G LTE or 5G NR systems, user data is transmitted on resource units [12], [13]. In the proposed scheme, we set the duration of OFDM and FTN symbols to be the same (i.e., $T_{\text{OFDM}} = T_{\text{FTN}}$), which simplifies the scheduling operation in the basestation as well as the buffering operation at the user terminal. Similarly, alignment of resource block (RB)
between OFDM and FTN symbols can also simplify the system operation by enabling the transceiver processing per RB. Towards this end, the squeezing factor $\beta$ should be chosen such that the number of subcarriers for FTN and OFDM in one RB satisfies $N_{RB,OFDM} \geq N_{RB,FTN}$, given that $\Delta f_{FTN} = \beta \Delta f_{OFDM}$.

Fig. 3 shows the case where each RB consists of 12 subcarriers (for the OFDM signaling) and 15 and 20 subcarriers (for the FTN signaling). In general, downlink slot consists of control channel, pilot signal, and data. In case each slot consists of 14 symbols as in 5G NR system [1], two symbols are used for the control channel, one symbol is used for the pilot signaling, and the rest of 11 symbols are used for the data transmission. When compared to the OFDM systems, the proposed FTN transmission scheme can achieve up to 20% spectral efficiency gain for $\beta = 0.8$. In case the downlink slot consists only of pilot signals and data, the spectral efficiency gain is increased up to 23%.

### B. Practical Consideration on the Proposed Hybrid Signaling

In this subsection, we discuss the sampling rate alignment between OFDM and FTN symbols used to facilitate the implementation of the proposed FTN-based hybrid signaling. By the alignment of the sampling rate, complicated timing switch between OFDM and FTN signaling can be prevented. Fig. 4 depicts the multi-carrier signal generation procedure at the transmitter. Without loss of generality, the IFFT size is set to be larger than or equal to the number of data subcarriers $N_d$ (i.e., $N = 2^m \geq N_d$ ($m \in \mathbb{Z}$)). After the IFFT operation, oversampling (with factor of $M$) and digital-to-analog conversion (DAC) with the sampling rate $T_s$ are performed. Finally, under given symbol duration $T$, pulse shaping (e.g., square-root raised cosine) is applied to obtain the time-domain samples in the analog domain.

Two factors determining the alignment of the sampling rate $T_s$ between OFDM and FTN symbols are the IFFT size $N$ and oversampling factor $M$ (see Fig. 4). First, in order to avoid the change of the IFFT size in the same slot, the IFFT size of the OFDM signaling should be set such that $N \geq N_{d,FTN}$ ($N_{d,FTN} > N_{d,OFDM}$) where $N_{d,FTN}$ and $N_{d,OFDM}$ are the number of data subcarriers supported by the FTN and OFDM signaling, respectively. For example, as observed from Table I, when the system bandwidth is 15 MHz and $\Delta f = 15$ kHz, OFDM can use 1024 IFFT size for $N_{d,OFDM} = 948$, but FTN uses 2048 IFFT size for $N_{d,FTN} = 1185$. By choosing the IFFT size $N = 2048$, we can employ the same IFFT size in the transmission time interval.

We next investigate the oversampling factor $M$ with the same IFFT size $N$ for both OFDM and FTN signaling. Note that the sampling rates of the OFDM and FTN symbols are expressed as $T_{s,OFDM} = \frac{1}{\Delta f_{OFDM} M_{OFDM}}$ and $T_{s,FTN} = \frac{1}{\Delta f_{FTN} M_{FTN}}$, respectively. In order to avoid the switching of the sampling rate in the same slot, we need to choose the oversampling factor such that $\beta M_{FTN} = M_{OFDM}$. As shown in Fig. 5, by choosing the oversampling factors $M_{OFDM} = 4$ and $M_{FTN} = 5$ when $\beta = 0.8$, we can satisfy the alignment condition $\beta M_{FTN} = M_{OFDM}$.

In summary, two conditions to align the sampling timing of OFDM and FTN symbols are as follows:

- The IFFT size $N$ for both OFDM and FTN signaling should satisfy $N \geq N_{d,FTN}$.
- The oversampling factors for OFDM and FTN should satisfy $\beta M_{FTN} = M_{OFDM}$.

### C. PAPR Analysis

In this subsection, we investigate the peak to average power ratio (PAPR) of the proposed FTN-based hybrid signaling scheme. Note that the PAPR is an important metric in the waveform design since the high PAPR brings on the signal distortion in the nonlinear region of the high power amplifier [14]. The PAPR of continuous-time signal $x(t)$ is defined as

$$PAPR = \max_{t} \frac{|x(t)|^2}{E[|x(t)|^2]},$$

(7)
where \( |x(t)|^2 \) is the peak power of the signal \( x(t) \) and \( E[|x(t)|^2] \) is the average power. The discrete version of PAPR can be expressed as

\[
PAPR = \frac{\max_{k \in \{0, \ldots, Q-1\}} |x[k]|^2}{E[|x[k]|^2]} \tag{8}
\]

where \( Q \) is the number of time-domain samples at a given symbol duration \( T \) (see (3)). When the number of subcarriers is sufficiently large, the distribution of \( |x[k]|^2 \) approaches the complex Gaussian and thus \( |x[k]|^2 \) can be readily approximated to \( \chi^2 \)-random variable with 2 degrees of freedom. Thus, the PDF is given by \( p_X(x) = \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}} \) and the PDF of \( Y = |x_0[k]|^2 = \frac{|x[k]|^2}{E[|x[k]|^2]} \) is \( p_Y(y) = 2\sigma^2 p_X(2\sigma^2 y) = e^{-y} \).

Since the CDF of \( Y \) is given by \( P_r(Y \leq y) = \int_0^y p_Y(y) = 1 - e^{-y} \), the CDF of PAPR in (8) is

\[
P_r(PAPR \leq \gamma) = P_r \left( \bigcap_{i=0}^{Q-1} \{|x_0[k]|^2 \leq \gamma\} \right) = \prod_{i=0}^{Q-1} P_r \left( \{|x_0[k]|^2 \leq \gamma\} \right) = \prod_{i=0}^{Q-1} \left( 1 - e^{-\gamma} \right), \tag{9}
\]

and hence the complementary CDF of the PAPR is

\[
P_r(PAPR > \gamma) = 1 - \prod_{i=0}^{Q-1} \left( 1 - e^{-\gamma} \right) = 1 - \left( 1 - e^{-\gamma} \right)^Q. \tag{10}
\]

When the IFFT sizes of OFDM and FTN symbols are the same and the oversampling factors satisfy \( \beta M_{FTN} = M_{OFDM} \), the number of time domain samples \( Q \) in the same symbol duration \( T \) equals for both OFDM and FTN symbols. It is not hard to convince from this discussion that when the OFDM and FTN symbols have the same number of time domain samples \( Q \), similar PAPR performance can be achieved for both OFDM and FTN symbols (see Fig. 6).

\[\text{D. Transmission Capacity}\]

In this subsection, we compare the capacity of the conventional OFDM signaling and FTN-based signaling schemes.

The achievable capacity for the OFDM signaling and FTN-based signaling under SISO AWGN channel can be expressed as

\[
C_{\text{OFDM}} \approx W \log_2(1 + \frac{P_S}{P_N}),
\]

\[
C_{\text{FTN}} \approx W \frac{1}{\beta} \log_2(1 + \frac{P_S}{P_N + P_{CI}}),
\]

where \( W \) is the signal bandwidth, \( P_S \) is the signal power, \( P_N \) is the noise power, and \( P_{CI} \) is the ICI power. One can easily see that the maximum achievable capacity of the FTN-based signaling is higher than that of the OFDM signaling when ICI is controlled properly. When we set \( \beta = 0.8 \), performance gap of the proposed scheme over the OFDM signaling with perfect channel state information is negligible [4]. In Fig. 7, we observe that the capacity gain of the FTN-based signaling over the OFDM signaling is about 23\%, which is close to the maximum achievable capacity gain (i.e., 25\%) for \( \beta = 0.8 \).

\[\text{IV. SIMULATION RESULTS AND DISCUSSIONS}\]

\[\text{A. Simulation Setup}\]

In this section, we evaluate the downlink performance of the proposed FTN-based hybrid signaling scheme. In the simulations, we set \( \beta = 0.8 \) and thus consider a time-frequency RB occupying 12 and 15 frequency subcarriers for one OFDM symbol and FTN symbol and 14 time symbols in one slot. Under the condition that 2 symbols are used for the control channel, one symbol is used for the pilot signaling and the remaining 11 symbols are used for the data transmission.

For the performance comparison, mean square error (MSE) of the channel estimation and the average throughput per RB (i.e., successfully transmitted bits per RB) are evaluated. The MSE is defined as

\[
MSE = E \left[ \text{tr} \left( [\hat{h} - h] [\hat{h} - h]^H \right) \right]. \tag{11}
\]

We assume the block Rayleigh fading channel where the channel gain remains constant within a processing block. As a receiver technique for mitigating ICI terms, we employ the BCJR equalization [8]. Let \( N_{p,OFDM}, N_{p,FTN}, N_{p,FTN-m} \) be the number of pilot symbols in one symbol for OFDM,
FTN, and FTN with mutually orthogonal pilots, respectively. The pilot symbol allocation in a specific symbol is set to $N_{p,\text{OFDM}} = 6$, $N_{p,\text{FTN}} = 7$, by following comb pilot symbol pattern [1]. For the case of FTN with mutually orthogonal pilots, we set $N_{p,\text{FTN-m}} = 3$ to satisfy the mutual orthogonality among pilot subcarriers. For the proposed signaling scheme, the same number of $N_{p,\text{OFDM}} = 6$ pilots is served by the OFDM signaling during the channel estimation process.

### B. Simulation Results

The MSE performance of $2 \times 2$ MIMO systems is provided in Fig. 8. The MSE of the channel estimates in OFDM symbol depends solely on the power spectral density $N_0$. Note, however, the MSE of the FTN signaling is affected by the amount of ICI between pilots from the same antenna as well as different antenna ports. We observe that the FTN with mutually orthogonal pilots outperforms the system employing only FTN signaling due to the mutually orthogonal pilot symbols. However, the FTN with orthogonal pilot mapping still suffers from ICI caused by the other antenna port. Since the OFDM signaling and the proposed hybrid signaling guarantee the orthogonality among pilot subcarriers from the same antenna port and the different antenna port, they show the best MSE performance at all SNR regime.

In Fig. 9, the throughput of $2 \times 2$ MIMO systems is provided. In this simulation, we set the QPSK modulation and the maximum available bits per RB for the OFDM and FTN-based schemes to be 528 and 660, respectively. Since the symbol used in the channel estimation process contains the pilot subcarriers only in all cases, the performance gap among the FTN-based schemes solely comes from the difference in the channel estimation quality. We observe from Fig. 9 that the spectral efficiency gain can be achieved by exploiting larger number of subcarriers with proper ICI control at the receiver.

### V. Conclusion

In this paper, we proposed the FTN signaling scheme suitable for the MIMO transmission. Our work was motivated by the observation that the FTN-based pilot transmission degrades the channel estimation quality severely. Key features of the proposed scheme are 1) channel estimation using the OFDM signaling and 2) data reception using the FTN signaling. We observed from numerical evaluations that the proposed hybrid scheme achieves a substantial gain in the spectral efficiency over the OFDM and FTN systems. One future direction is to improve the channel estimation quality of the proposed FTN signaling by recycling the detected symbols as pilots [15].

### REFERENCES

[1] 3GPP TS 38.211, “Physical channels and modulation (Release 15),” v15.2.0, Jun. 2018.