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Enhanced Sparse Vector Coding for Ultra-Reliable and Low Latency Communications

Wonjun Kim, Shravan Kumar Bandari, and Byonghyo Shim
Institute of New Media and Communications and Department of Electrical and Computer Engineering,
Seoul National University, Seoul, Korea

Abstract

An important observation in the ultra-reliable and low latency communications is that the size of transmit information is tiny. To support the effective short packet transmission, a sparse vector coding (SVC) scheme where an information is encoded into the positions of the sparse vector was proposed. In this paper, we propose a novel SVC technique further improving the reliability of the short packet transmission. Key idea of the proposed technique is to encode information both in the position as well as symbols. From the performance analysis and numerical evaluations on realistic channel models, we demonstrate that the proposed scheme outperforms the conventional SVC scheme in terms of the block error rate (BLER) and transmission latency.

I. INTRODUCTION

Fifth generation (5G) wireless communication needs to support variety of use cases such as ultra-reliable and low-latency communications (URLLC), enhanced mobile broadband (eMBB),
and massive machine-type communication (mMTC) [1], [2]. Among these, the URLLC is perhaps most challenging use case since it should satisfy two stringent requirements simultaneously: latency and reliability [3]. One notable observation in many URLLC applications such as autonomous vehicles, remote emergency surgery, and smart factory, is that the transmit information is control or command type information (e.g., move left/right, open/close, start/stop, rotate/shift, and speed up/down) or sensing information (e.g., temperature, moisture, pressure, and gas density) [3], [4] so that an amount of information to be delivered is very tiny (10~100 bits). For this reason, today’s wireless transmission paradigm, designed to maximize the data throughput by transmitting a long packet, is an overkill for the URLLC scenarios [5]. Without doubt, how to deal with the short packet transmission is an important issue in URLLC.

Recently an approach to transmit the short-sized packet for the URLLC scenario, referred to as sparse vector coding (SVC), has been proposed [6]. Main idea of SVC is to encode information into the positions of a sparse vector. In a nutshell, SVC can be viewed as a transformation of the information into the sparse vector followed by the random spreading. The decoding of SVC can be formulated as a sparse recovery problem in compressed sensing (CS). It has been shown that SVC outperforms the 4G LTE and 5G NR Physical Downlink Control Channel (PDCCH) in many realistic URLLC scenarios.

An aim of this paper is to propose a novel SVC technique further improving the reliability of the short packet transmission. The main idea of the proposed scheme, henceforth referred to as enhanced sparse vector coding (ESVC), is to embed part of the information bits to the sparse vector and the rest in a form of symbols at the chosen nonzero positions. While SVC encodes information only in the position of a sparse vector, ESVC encodes information both in the position as well as symbol\(^1\). In doing so, dimension of the sparse vector can be made smaller than that of the conventional SVC. It is now well known from the principle of CS that when the dimension of a sparse vector decreases, required number of measurements (resources) ensuring an accurate sparse recovery will also decrease [8]. In our context, this implies that under the condition that the same amount of resources is used, the system model becomes

\(^1\)We note that the ESVC scheme is similar in spirit to the index modulation (IM) in the sense that the information bit is encoded into the position as well as symbol in both approaches [7]. However, ESVC is distinct from the IM since the information bit in IM is encoded into the indices of active subcarriers in IM, while it is mapped into a sparse vector and then compressed into the resources using the pseudo-random spreading sequence in ESVC. Therefore, sparse recovery algorithm in compressed sensing is used in the decoding (see Section II.B).
less underdetermined and thus columns of the sensing matrix become less correlated, resulting in a considerable improvement in the decoding performance. From the performance analysis and numerical evaluations, we demonstrate that the proposed ESVC scheme outperforms the conventional SVC by a large margin. For example, we observe that ESVC achieves around 2 dB gain over SVC in the extended pedestrian-A (EPA) and extended vehicular-A (EVA) channel models.

II. ENHANCED SPARSE VECTOR CODING

A. Basics of SVC

Consider the situation where a user transmits $b_i$ bits of information using $m$ physical resources. In the first step of SVC, an information vector is mapped to the sparse vector $s$. Let $k$ be the sparsity (the number of nonzero elements) and $N$ be the length of the sparse vector, then we have $\binom{N}{k}$ choices to generate $k$-sparse vector so that we can encode $\lceil \log_2 \binom{N}{k} \rceil$ bits of information. As an example, if we pick two nonzero positions in 36-bit vector (i.e., $k = 2$ and $N = 36$), then we have 630 choices in total and hence 9 bits of information can be encoded by SVC (see Fig. 1(a)). In the second step, each nonzero element of the sparse vector $s$ is spread into $m$ resources ($m \ll N$) using a pseudo-random spreading. The codebook matrix $C = [c_1 \ldots c_N]$ is designed such that the spread vector $Cs$ contains enough information to recover the input sparse vector $s$ regardless of the choice of nonzero positions. The received vector can be expressed as [6],

$$y = HCs + v,$$

where $H = \text{diag}(h)^2$ and $v \sim \mathcal{CN}(0, \sigma_v^2 I)$ is the additive white Gaussian noise (AWGN). Since the decoding process is done by the support identification, a sparse recovery algorithm such as orthogonal matching pursuit (OMP) can be used.

B. Enhanced SVC

Fig. 2 depicts the block diagram of the proposed ESVC scheme. In the ESVC scheme, the transmit information is conveyed by using the support and symbols in the nonzero positions of

\footnote{\(h = [h_1, \ldots, h_m]^T\) is the channel vector.}

\footnote{If \(s = [0 \ 1 \ 0 \ 0 \ 1 \ 0]\), then the support is \(\Omega = \{2, 5\}\)}
s. To be specific, total number of bits $b_t$ is divided into $b_i$ and $b_m$ bits. Then, $b_i$ bit of information is encoded into the positions of sparse vector and $b_m$ bit of information is encoded in a form of symbol in the support ($b_m = kb_s$ where $b_s$ is bits per symbol). For example, when $b_i = 5$, $b_m = 4$, $k = 2$, and $\Omega = \{3,7\}$, the ESVC-encoded sparse vector is $\tilde{s} = [0 \ 0 \ \tilde{s}_3 \ 0 \ 0 \ \tilde{s}_7 \ 0 \ 0]^T$ where $\tilde{s}_3$ and $\tilde{s}_7$ are the QPSK modulated symbols (see Fig. 1(b)). After the sparse vector mapping, the spreading codebook $\tilde{C}$ is applied to $\tilde{s}$ and then transmitted. The corresponding received signal $y$ is

$$y = H\tilde{C}\tilde{s} + v = \Phi\tilde{s} + v,$$

(2)

where $\Phi = H\tilde{C}$ is the sensing matrix.

We briefly discuss the key factor bringing the performance gain of ESVC over the conventional SVC technique. We denote the dimension of sparse vectors for SVC and ESVC as $N_1$ and $N_2$ respectively. Given the total number of bits $b_t$ and sparsity $k$, $N_1$ is determined by the relationship $b_t = \left\lfloor \log_2 \left( \frac{N_1}{k} \right) \right\rfloor$. In a similar way, $N_2$ is obtained from $b_t - kb_s = \left\lfloor \log_2 \left( \frac{N_2}{k} \right) \right\rfloor$. Since $kb_s$ is a positive integer, one can see that $N_2$ is smaller than $N_1$ ($N_2 < N_1$). Based on the principle of CS, an accurate recovery of the sparse vector is possible as long as $m \approx ck \log N$ where $c$ is the properly chosen constant [8]. Under the same set of conditions, therefore, the required number of measurements of ESVC is smaller than that of SVC. Alternatively, if the number of measurements is the same, the underdetermined ratio $\frac{N_2}{m}$ of the ESVC-encoded vector becomes...
smaller than that of SVC-encoded vector (i.e., $\frac{N_1}{m} > \frac{N_2}{m}$), so that the mutual coherence\(^4\) of $\Phi$ will be reduced.

Since the information is encoded in both support and symbol, decoding process of ESVC process is divided into two steps: 1) support identification and 2) symbol detection. In the first step, we need to find out $k$ nonzero positions of $\tilde{s}$. To this end, a sparse recovery algorithm such as OMP can be employed \[^9\]. Main task of OMP is to identify a column of $\Phi$ that is maximally correlated with the (modified) observation $r^{j-1}$ in each iteration. That is, an index of the nonzero column of $\Phi$ chosen in the $j$-th iteration is given by\(^5\)

$$\omega_j = \arg \max_{1 \leq n \leq N} |\langle \phi_n, r^{j-1} \rangle|^2,$$

where $r^{j-1} = y - \Phi_{\Omega_s} \hat{s}^{j-1}$ is the modified observation called the residual and $\hat{s}^{j-1} = (\Phi_{\Omega_s}^T \Phi_{\Omega_s})^{-1} \Phi_{\Omega_s}^T y$ is the estimate of $\tilde{s}$ at $(j-1)$-th iteration\(^6\). Once the support is identified, we can convert the underdetermined system into the overdetermined system. For example, if $\Omega_s = \{2, 5\}$, then

\(^4\)The mutual coherence $\mu^*$ is defined as the largest magnitude of column correlation in the sensing matrix (i.e., $\mu^* = \max_{i \neq j} |\langle \phi_i, \phi_j \rangle|$)

\(^5\)If $\Omega_s = \{2, 5\}$, then $\Phi_{\Omega_s} = [\phi_2, \phi_5]$.

\(^6\) $\Phi^T = (\Phi \Phi^T)^{-1} \Phi^T$ is the pseudo-inverse of $\Phi$
the system model in (2) is simplified to \( \mathbf{y} = \begin{bmatrix} \phi_2 & \phi_5 \end{bmatrix} \begin{bmatrix} \tilde{s}_2 \\ \tilde{s}_5 \end{bmatrix} + \mathbf{v} \). Then, in the second step, conventional technique such as the minimum mean square error (MMSE) detector can be used for the symbol detection.

III. ESVC PERFORMANCE ANALYSIS

In this section, we present the block error rate (BLER) analysis of the ESVC technique. For the fair comparison with the conventional SVC, we set \( k = 2 \) and \( \Omega_{\tilde{s}} = \{p, q\} \) (i.e., \( \tilde{s}_\Omega = [\tilde{s}_p, \tilde{s}_q]^T \) where \( \tilde{s}_p = \Re_p + j\Im_p \) and \( \tilde{s}_q = \Re_q + j\Im_q \)). In essence, block error occurs due to the following two cases:

1) The sparse recovery algorithm fails to detect the correct support (i.e., \( \Omega^*_\tilde{s} \neq \Omega_{\tilde{s}} \)).

2) Maximum likelihood (ML) detector fails to find out the symbols correctly after the successful support identification (i.e., \( \mathbf{s}^* \Omega = \tilde{s}_{\Omega} \)).

Using these, the BLER of the ESVC-encoded packet can be expressed as

\[
\text{BLER} = P \left( \{ \Omega^*_\tilde{s} \neq \Omega_{\tilde{s}} \} \cup \{ \mathbf{s}^* \Omega \neq \tilde{s}_{\Omega} | \Omega^*_\tilde{s} = \Omega_{\tilde{s}} \} \right) \\
\leq P (\Omega^*_\tilde{s} \neq \Omega_{\tilde{s}}) + P (\mathbf{s}^* \Omega 
eq \tilde{s}_{\Omega} | \Omega^*_\tilde{s} = \Omega_{\tilde{s}}) \tag{4}
\]

where \((a)\) follows from the union bound of probability theory. Our main result is as follows.

**Theorem 1**: The block error rate of the ESVC packet can be upper bounded as,

\[
\text{BLER} \leq \left[ 1 - \left( 1 - \left( 1 + \frac{\alpha_1}{2\sigma_v^2} \right)^{-m} \left( 1 + \frac{\beta_1}{\sigma_v^2} \right)^{-m} \right)^{N-1} \right] \\
\times \left[ 1 - \left( 1 + \frac{\alpha_2}{2\sigma_v^2} \right)^{-m} \left( 1 + \frac{\beta_2}{\sigma_v^2} \right)^{-m} \right]^{N-2} \\
+ k \left( 1 + \frac{\mu^* d_{\text{min}}^2}{4\sigma_v^2} \right)^{-m} \tag{5}
\]

where \( m \) is the number of measurements (resources), \( k \) is the sparsity, \( \sigma_v^2 \) is the noise variance, \( \mu^* = \max_{i \neq j} |\mu_{ij}| \) is the absolute value of the maximum correlation between two columns of \( \Phi \), \( d_{\text{min}} \) is the minimum euclidean distance between the symbols, and \( \alpha_1 = \{\Re_p + \Re_q \mu^* - (|\Re_p| + |\Re_q|) \mu^* \}^2 \), \( \beta_1 = \{\Re_p + \Re_q \mu^* \}^2 \), \( \alpha_2 = \{|\Im_q| - |\Im_q| \mu^* \}^2 \), \( \beta_2 = \Im_q^2 \).

\(\Re_i\) and \(\Im_i\) are the real and imaginary parts of the \( i \)-th symbol, respectively.
Following lemmas will be useful to prove Theorem 1. **Lemma 1**: The probability that the support elements are chosen correctly satisfies

\[ P_{\text{succ}} \geq \left( 1 - \left( 1 + \frac{\alpha_1}{2\sigma_v^2} \right)^{-m} - \left( 1 + \frac{\beta_1}{\sigma_v^2} \right)^{-m} \right)^{N-1} \]
\[ \times \left( 1 - \left( 1 + \frac{\alpha_2}{2\sigma_v^2} \right)^{-m} - \left( 1 + \frac{\beta_2}{\sigma_v^2} \right)^{-m} \right)^{N-2} \].

(6)

**Proof**: Let \( S^j \) be the success probability of the chosen support element in the \( j^{th} \) iteration, then the probability that the support elements are correctly chosen can be expressed as

\[ P_{\text{succ}} = P(\Omega_\delta^* = \Omega_{\delta}) = P(S^1, S^2) = P(S^2|S^1) P(S^1). \]

For the analytic simplicity, we take real part of the decision statistics in the first iteration and then imaginary part in the second iteration. For a given channel realization \( h = [h_1, \cdots, h_m]^T \), to identify the support element in the first iteration, we should have

\[ \left| \Re \langle \phi_p^{\parallel} || h \|_2, r_0 \rangle \right| \geq \max_i \left| \Re \langle \phi_i^{\parallel} || h \|_2, r_0 \rangle \right|. \]

That is,

\[ P(S^1|h) = P \left( \left| \Re \langle \phi_p^{\parallel} || h \|_2, r_0 \rangle \right| \geq \max_i \left| \Re \langle \phi_i^{\parallel} || h \|_2, r_0 \rangle \right| \right) \]
\[ = \prod_{i=1,i\neq p}^N P \left( \left| \Re \langle \phi_p^{\parallel} || h \|_2, r_0 \rangle \right| \geq \left| \Re \langle \phi_i^{\parallel} || h \|_2, r_0 \rangle \right| \right) \].

(7)

Noting that \( s_p = \Re_p + j\Im_p \) and \( s_q = \Re_q + j\Im_q \) we have,

\[ \Re \langle \phi_p^{\parallel} || h \|_2, r_0 \rangle \]
\[ = \Re \langle \phi_p^{\parallel} || h \|_2, \phi_p s_p + \phi_q s_q + v \rangle \]
\[ = \Re \langle h ||_2, \mu_{pq} || h \|_2 + z_r, \]

(8)

where \( \mu_{ij} \) is the correlation between \( c_i \) and \( c_j \) and \( z_r = \Re \langle \phi_p^{\parallel} || v \rangle \). Similarly, we have

\[ \Re \langle \phi_i^{\parallel} || h \|_2, r_0 \rangle \]
\[ = \Re \langle h ||_2, \mu_{iq} || h \|_2 + z_i, \]

(9)
where $z_i = \Re \left( \frac{\phi_i}{\| \phi_i \|_2} v \right)$. Using (8), (9), and $P(|A| \geq |B|) = P(A > |B|)P(A > 0) + P(-A > |B|)P(A < 0)$, we have

$$P \left( \left| \Re \langle \phi_p, r^0 \rangle \right| \geq \left| \Re \langle \phi_i, r^0 \rangle \right| \right) = P \left( \| (\Re_p + \Re_q \mu_{pq}) \|_2 + z_r \| \geq \| (\Re_p \mu_p + \Re_q \mu_{iq}) \|_2 + z_i \right) \tag{10}$$

$$(a) \geq 1 - \exp \left( -\frac{\| h \|_2^2 \alpha_1}{2\sigma_v^2} \right) - \exp \left( -\frac{\| h \|_2^2 \beta_1}{\sigma_v^2} \right) \tag{11}$$

where (a) is because $P(z_p < -\| h \|_2) = 1 - Q \left( \frac{-\| h \|_2}{\sigma_v \sqrt{2}} \right) \geq 1 - \exp \left( -\frac{\| h \|_2^2}{2} \right)$. Then, taking the expectation with respect to the channel realization $h^8$, we have

$$P(S^1) = \int P(S^1|h)f_h(x)dx = E_h \left[ P(S^1|h) \right]$$

$$\geq \prod_{i=1, i \neq p}^N \left( 1 - E_h \left[ \exp \left( -\| h \|_2^2 \alpha_1 \right) \right] \right. \left. - E_h \left[ \exp \left( -\| h \|_2^2 \beta_1 \right) \right] \right) \tag{12}$$

$$\geq \left( 1 - \left( 1 + \frac{\alpha_1}{2\sigma_v^2} \right)^{-m} - \left( 1 + \frac{\beta_1}{\sigma_v^2} \right)^{-m} \right)^{N-1} \tag{13}$$

where (13) is because $E_h \left[ \exp \left( -\| h \|_2^2 \sigma_v^2 \right) \right] = \left( 1 + \frac{1}{\sigma_v^2} \right)^{-m}$.

In the second iteration, the success probability is given by

$$P(S^2|S^1) = P \left( \left| \Im \langle \phi_q, r^1 \rangle \right| \geq \max_i \left| \Im \langle \phi_i, r^1 \rangle \right| \right)$$

$$= \prod_{i=1, i \neq p, q}^N P \left( \left| \Im \langle \phi_i, r^1 \rangle \right| \geq \left| \Im \langle \phi_i, r^1 \rangle \right| \right) \tag{14}$$

where $r^1 = \phi_q s_q + v$. In a similar way, one can show that

$$P(S^2|S^1) \geq \left( 1 - \left( 1 + \frac{\alpha_2}{2\sigma_v^2} \right)^{-m} - \left( 1 + \frac{\beta_2}{\sigma_v^2} \right)^{-m} \right)^{N-2}. \tag{15}$$

In the following Lemma, we obtain the symbol error rate (SER) of ESVC.

$^8\| h \|_2^2$ is a Chi-squared distribution [6].
Lemma 2: The symbol error rate of ESVC satisfies

\[ \text{SER}_{\text{ESVC}} \leq k \left( 1 + \frac{\mu^* d_{\text{min}}^2}{4\sigma_v^2} \right)^{-m} \]  

where \( d_{\text{min}} \) is the minimum Euclidean distance between the symbols (i.e., \( d_{\text{min}} = \min_{i,j \in \Omega, i \neq j} \| \tilde{s}_i - \tilde{s}_j \|^2_2 \)).

Proof: Let \( \tilde{s}_p \) be the transmit symbol and \( \tilde{s}_p \) be the incorrectly detected symbol for \( \tilde{s}_p \). The pairwise error probability of choosing \( \tilde{s}_p \) incorrectly can be expressed as

\[ \Pr (\tilde{s}_p \rightarrow \tilde{s}_p|y, h) = Q \left( \frac{\sqrt{\| \Phi_i (\tilde{s}_i - \tilde{s}_j) \|^2_2}}{2\sigma_v} \right) \leq \exp \left( -\frac{\mu^* \| h \|^2 d_{\text{min}}^2}{4\sigma_v^2} \right). \]  

where \( Q(\cdot) \) is the Gaussian Q-function and \( (a) \) is because \( Q(x) \leq \exp \left( -\frac{x^2}{2} \right) \). The unconditional PEP can be obtained by taking the expectation with respect to the channel realization \( h \) as

\[ \Pr (\tilde{s}_i \rightarrow \tilde{s}_j) = E_h [\Pr (\tilde{s}_i \rightarrow \tilde{s}_j|y, h)] = E_h \left[ \exp \left( -\frac{\| h \|^2 \Delta^2 + \gamma \Delta^2}{4\sigma_v^2} \right) \right] = \left( 1 + \frac{\mu^* \| h \|^2 d_{\text{min}}^2}{4\sigma_v^2} \right)^{-m}. \]  

Using the union bound, the SER of ESVC can be upper bounded as

\[ \text{SER}_{\text{ESVC}} = \Pr (\tilde{s}_i \rightarrow \tilde{s}_j) \leq k \left( 1 + \frac{\mu^* d_{\text{min}}^2}{4\sigma_v^2} \right)^{-m}. \]  

By combining Lemma 1 and Lemma 2, we finally obtain (5). It is clear from (5) that the BLER increases when the number of measurement \( m \) is small and information vector \( \tilde{s} \) is less sparse (i.e., \( k \) is large), which matches well with our expectation.

IV. Simulation Results

In this section, we compare the BLER performance of the proposed ESVC scheme and conventional SVC scheme. To ensure the fair comparison, we set \( b_t = 16, k = 2, b_i = 12 \) and \( b_m = 4 \). In the codebook generation, we use a randomly generated Bernoulli sequences. As a channel model, we use AWGN and realistic ITU models (EPA channel and EVA channel) [10].

In Fig. 3, we evaluate the BLER performance of the proposed ESVC and conventional SVC scheme in the AWGN scenario. We observe that the proposed ESVC scheme outperforms the conventional SVC. For example, the proposed scheme achieves 1.5 dB gain over the SVC at BLER=10^{-4}. Also, we see that the BLER decreases sharply when the number of measurements \( m \) increases. For instance, if \( m \) is changed from 42 to 64, we obtain around 2 dB gain at BLER=10^{-4}. 
In Fig. 4, we plot the BLER performance of the ESVC scheme and competing schemes under the EVA and EPA channels. We observe that the proposed ESVC outperforms the conventional PDCCH and SVC scheme by a large margin. For example, the ESVC scheme achieves 6.5 dB gain over the conventional PDCCH at BLER = $10^{-3}$ under EVA channel and around 2 dB gain over the conventional SVC at BLER = $10^{-3}$ under EPA channel.

Next, we plot the BLER performance of ESVC using various $b_s$ ($b_s = 1$ for BPSK, $b_s = 2$ for QPSK, and $b_s = 3$ for 8PSK) in Fig. 5. We can clearly see that the ESVC performance depends highly on $b_s$ because there is a tradeoff between the support identification performance and the symbol detection performance. Specifically, when $b_s$ increases, $N$ can be reduced so that the support identification performance will be improved at the expense of the performance degradation in the symbol detection. In fact, the symbol detection performance of the low order modulation is better than that of the high order modulation but the support identification performance would be poor due to the increase in the sparse vector dimension $N$.

Finally, we evaluate the latency performance of the ESVC and SVC. In Fig. 6, we plot the
distribution of transmission latency which is defined as the time from the initial transmission to the successful packet decoding at the mobile terminal. From the result, we observe that the transmission latency of ESVC is much smaller (15% on average) than the SVC transmission latency.

V. Conclusion

In this paper, we presented an enhanced sparse vector coding (ESVC) scheme suitable for the short-packet transmission. The key idea behind the proposed scheme is to encode information into the positions of the sparse vector as well as the symbols. In doing so, the dimension of a sparse vector can be made small and thus we obtain the improvement in the sparse recovery performance. We demonstrated from the numerical evaluations that the proposed ESVC scheme outperforms SVC in terms of BLER and transmission latency.
Fig. 5: BLER performance as a function of SNR for various $b_s$ ($b_t = 20$, $m = 50$, and $k = 2$).

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Fig. 6: Probability of transmission latency to complete the packet transmission ($b_t = 16$, $m = 42$ and SNR = $-2$ dB).


